Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/18-1.1.1.7-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [35]. This is test number [18].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (35)	0.00 (0)
Mathematica	100.00 (35)	0.00 (0)
Maple	100.00 (35)	0.00 (0)
Fricas	25.71 (9)	74.29 (26)
Mupad	0.00 (0)	100.00 (35)
Giac	0.00 (0)	100.00 (35)
Maxima	0.00 (0)	100.00 (35)
Sympy	0.00 (0)	100.00 (35)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

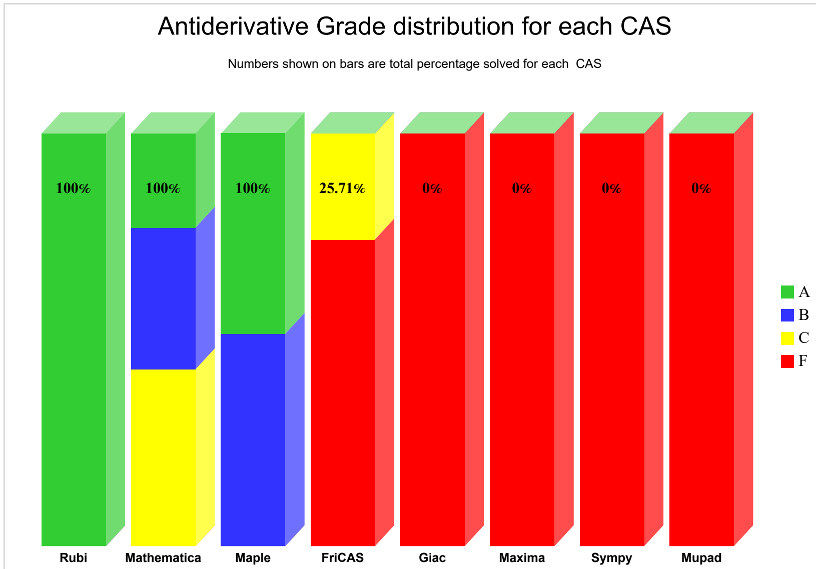
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

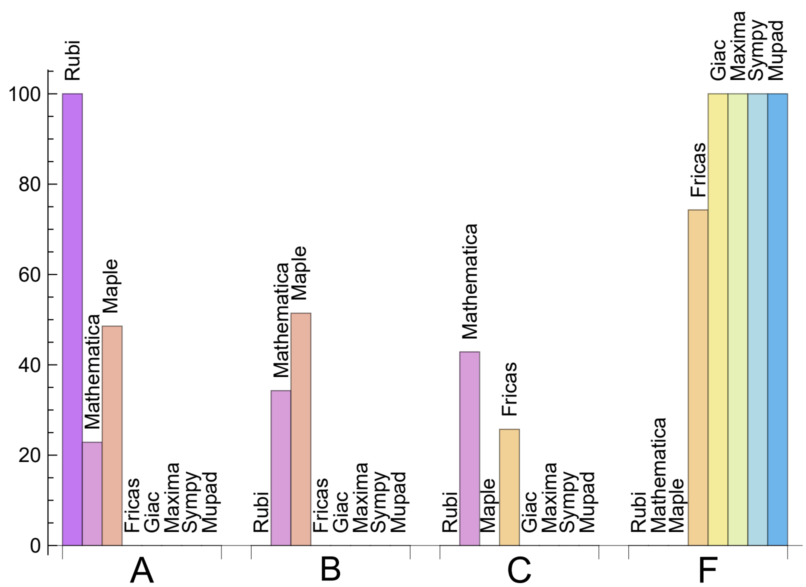
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	48.571	51.429	0.000	0.000
Mathematica	22.857	34.286	42.857	0.000
Fricas	0.000	0.000	25.714	74.286
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	26	26.92	73.08	0.00
Mupad	35	0.00	100.00	0.00
Giac	35	97.14	0.00	2.86
Maxima	35	100.00	0.00	0.00
Sympy	35	74.29	25.71	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.14
Rubi	1.14
Maple	5.25
Mathematica	31.13
Sympy	-nan(ind)
Maxima	-nan(ind)
Giac	-nan(ind)
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	694.46	1.00	680.00	1.00
Fricas	1007.22	1.98	859.00	2.08
Maple	1504.14	2.10	1238.00	1.85
Mathematica	5806.66	6.15	825.00	1.34
Sympy	-nan(ind)	-nan(ind)	nan	nan
Maxima	-nan(ind)	-nan(ind)	nan	nan
Giac	-nan(ind)	-nan(ind)	nan	nan
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

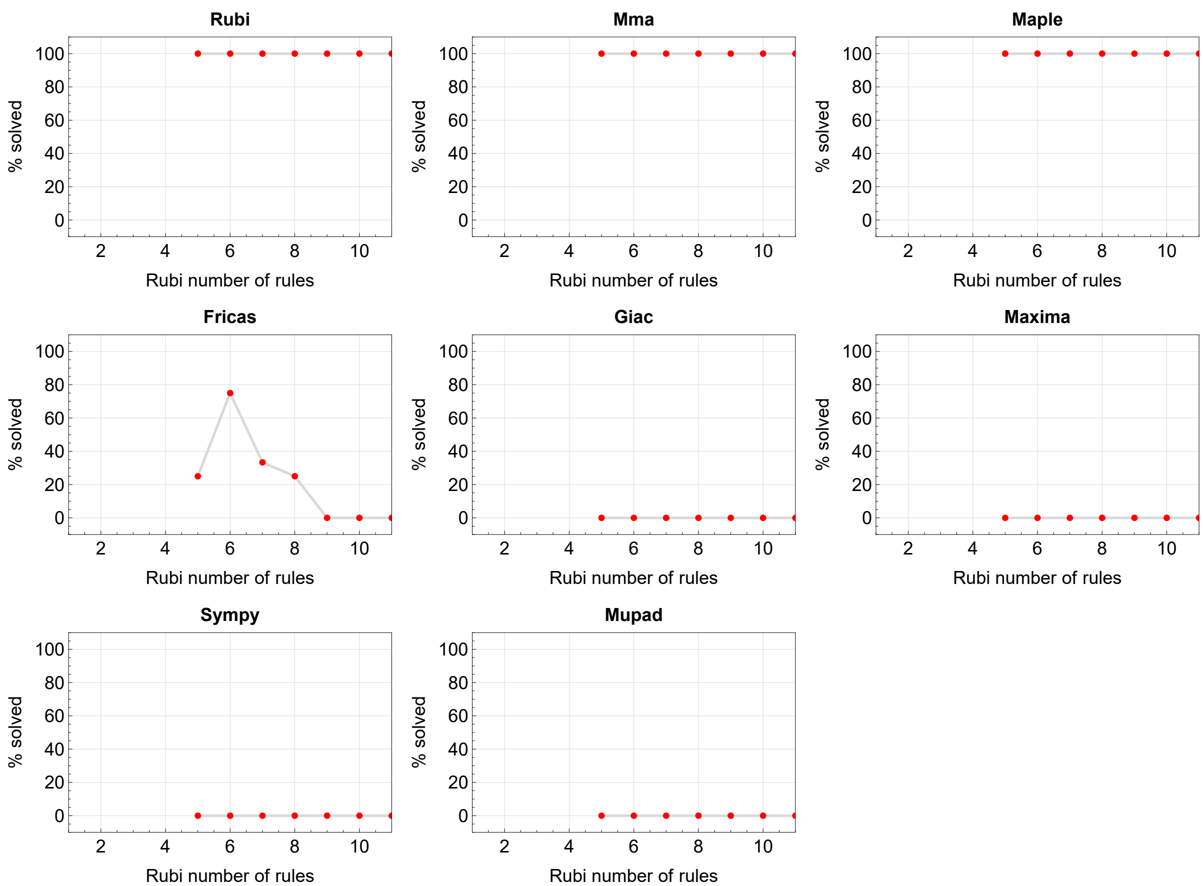


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

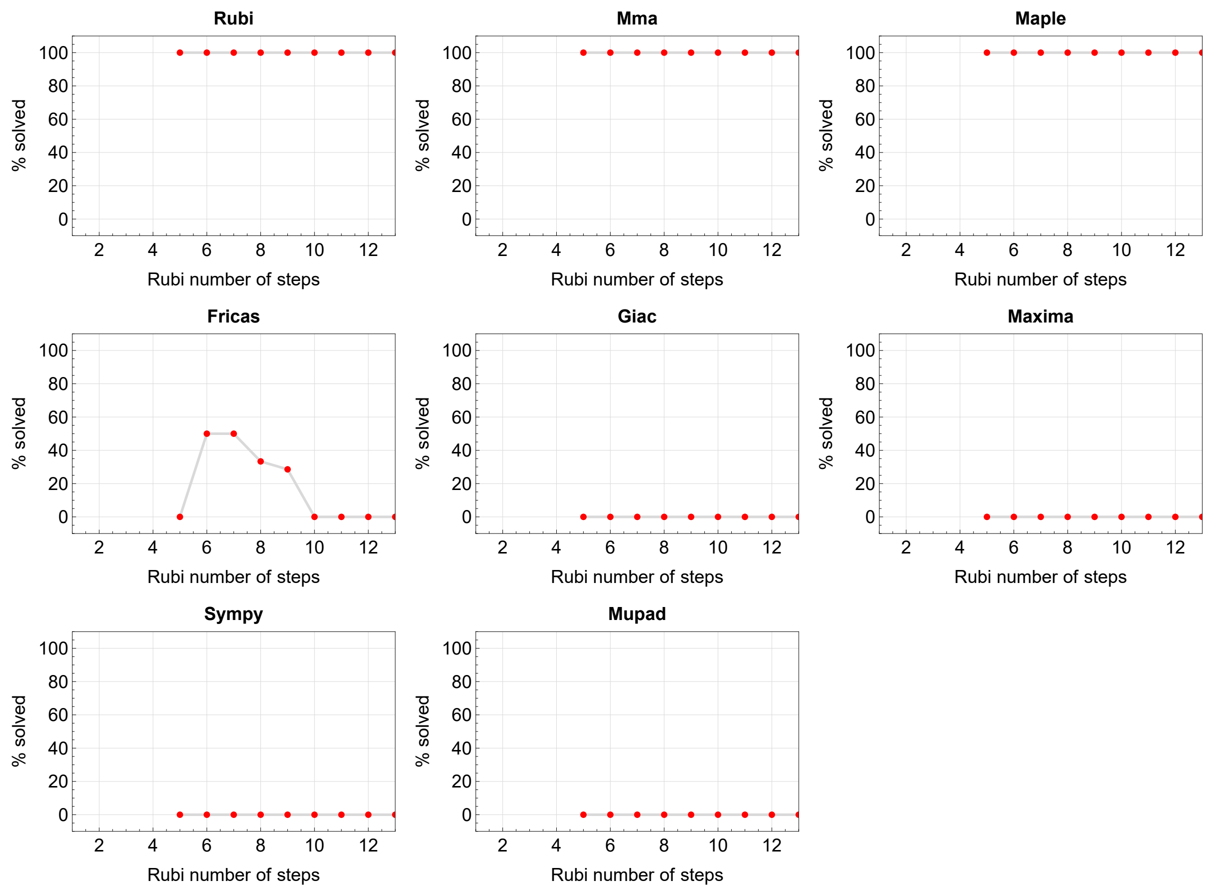


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

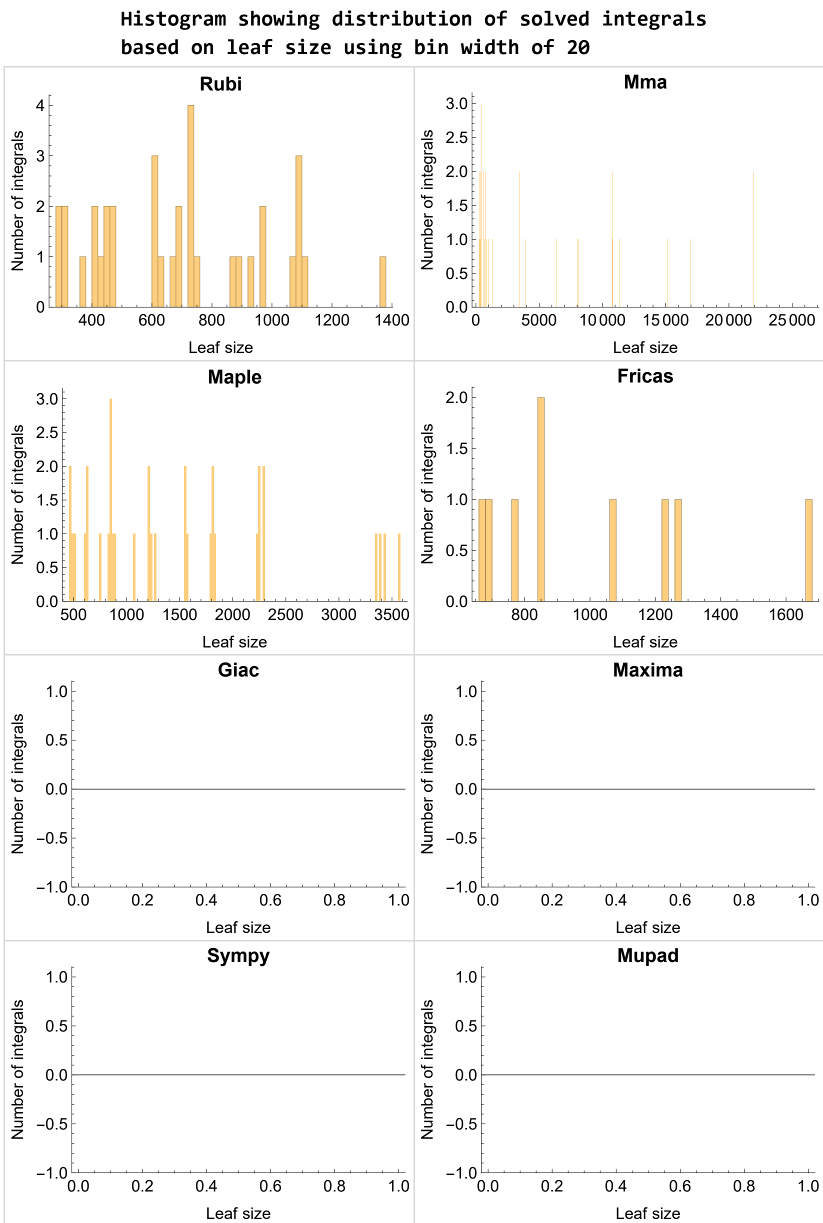


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

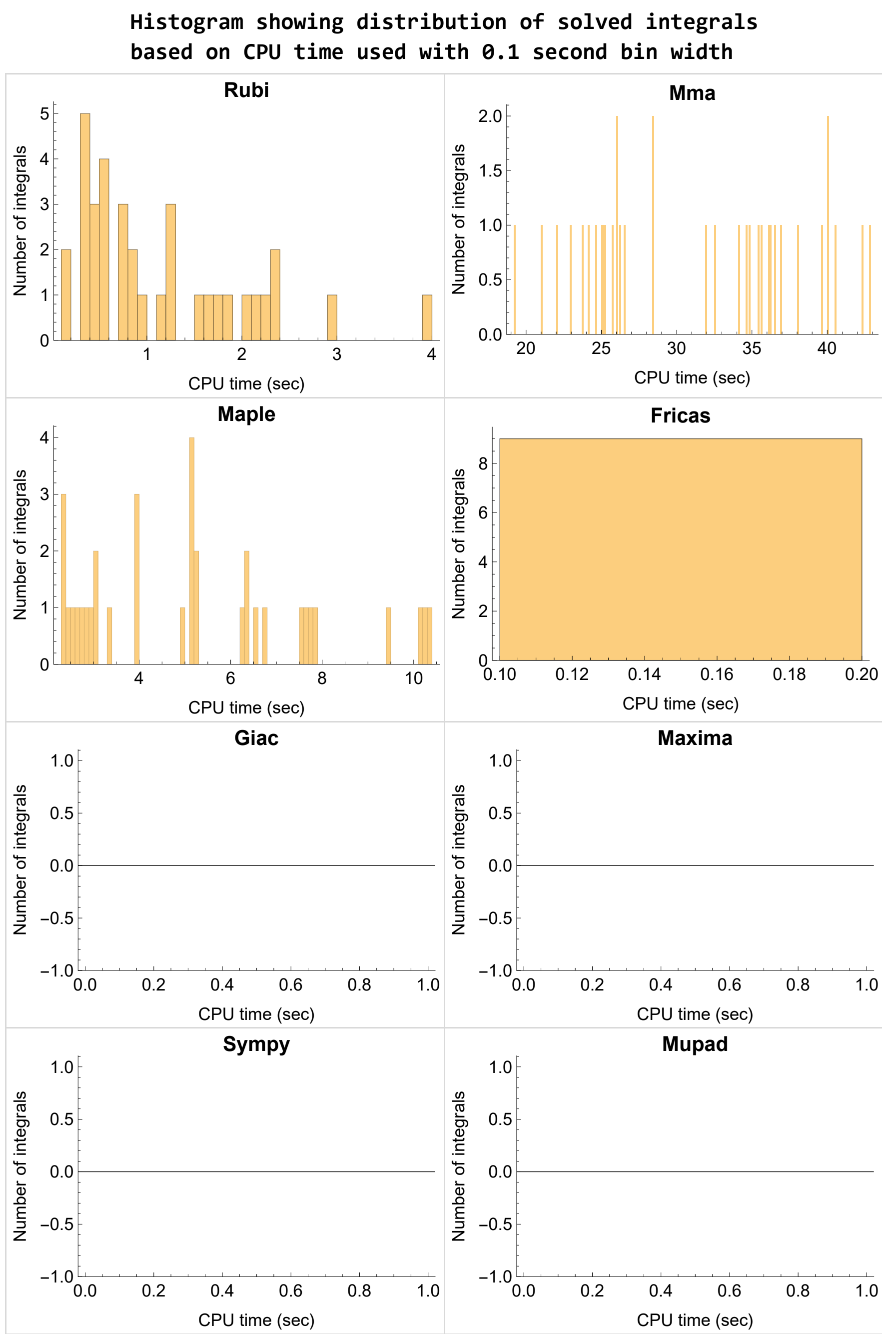


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

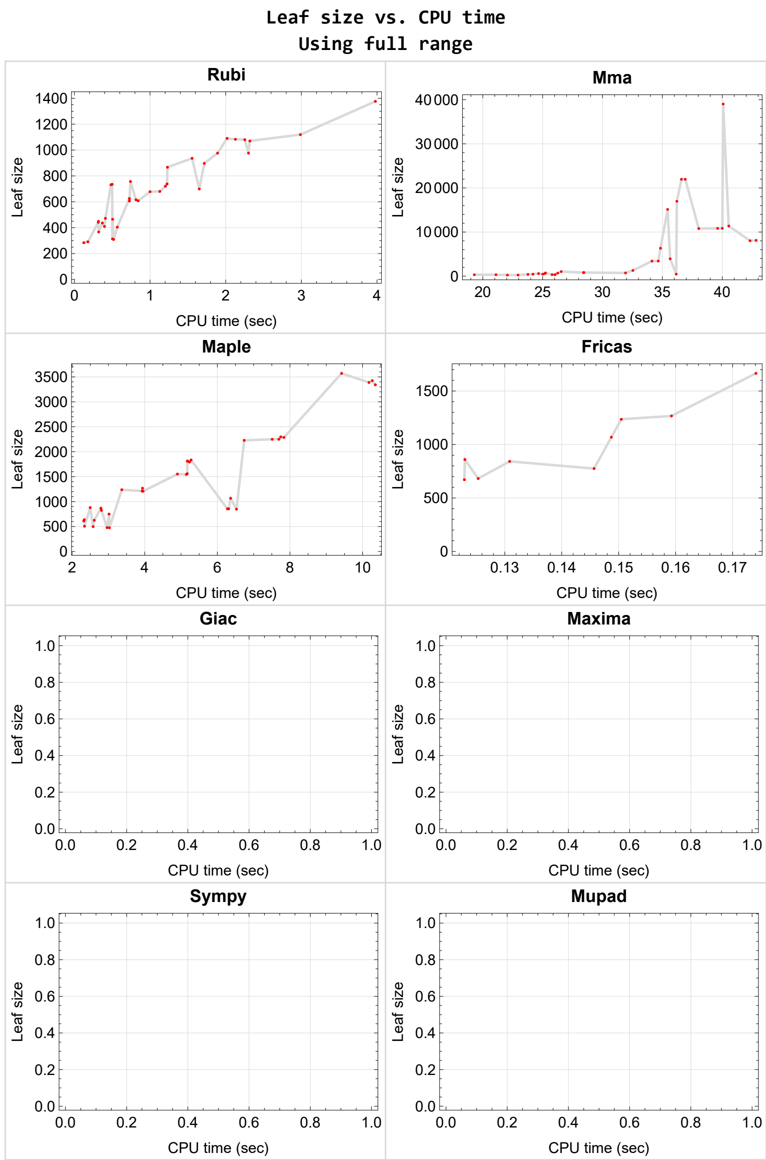


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{6, 10, 11, 21, 25, 31, 32, 34, 35\}$

Mathematica $\{6, 7, 11, 21, 22, 31, 32, 34\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

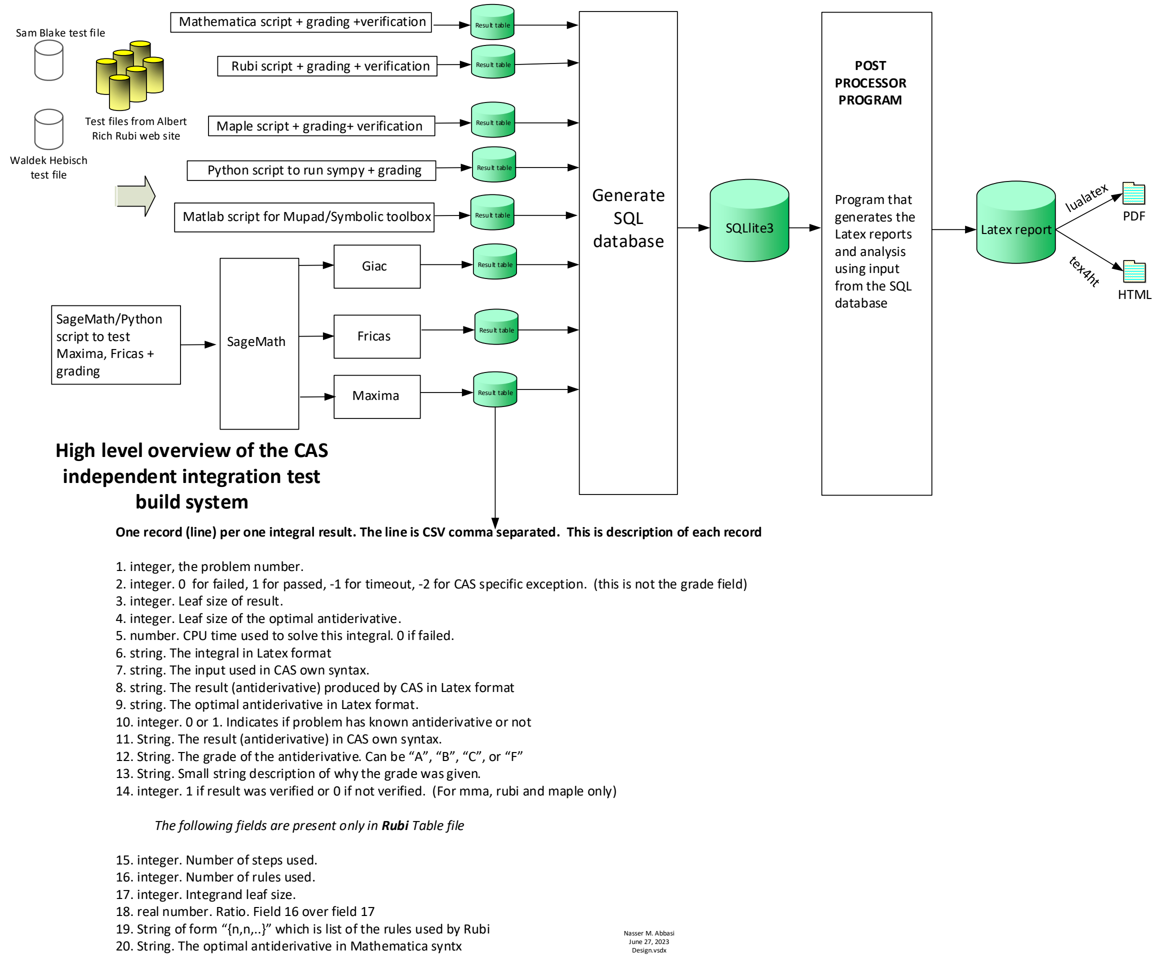
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 8, 9, 12, 13, 14, 23, 24, 34 }

B grade { 6, 7, 10, 11, 15, 21, 22, 25, 31, 32, 33, 35 }

C grade { 1, 2, 3, 4, 5, 16, 17, 18, 19, 20, 26, 27, 28, 29, 30 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 16, 17, 18, 19, 20, 26, 27, 28, 29, 30, 31, 33 }

B grade { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21, 22, 23, 24, 25, 32, 34, 35 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { }

B grade { }

C grade { 1, 2, 3, 16, 17, 18, 26, 27, 28 }

F normal fail { 9, 10, 14, 15, 24, 25, 35 }

F(-1) timedout fail { 4, 5, 6, 7, 8, 11, 12, 13, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35 }

F(-1) timedout fail { }

F(-2) exception fail { 34 }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 26, 27, 28, 29, 31, 32, 33, 34 }

F(-1) timedout fail { 5, 10, 15, 19, 20, 24, 25, 30, 35 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	700	699	806	866	0	1236	0	0	0
N.S.	1	1.00	1.15	1.24	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	1.651	28.405	2.793	0.000	0.150	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	404	450	625	0	842	0	0	0
N.S.	1	1.00	1.11	1.54	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.567	24.188	2.610	0.000	0.131	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.124	19.292	2.578	0.000	0.123	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	245	478	0	0	0	0	0
N.S.	1	1.00	0.78	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	22.947	2.962	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	678	678	3412	1208	0	0	0	0	0
N.S.	1	1.00	5.03	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.000	34.128	3.956	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	981	976	21961	1814	0	0	0	0	0
N.S.	1	0.99	22.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.301	36.590	5.178	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	736	735	8030	1544	0	0	0	0	0
N.S.	1	1.00	10.91	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	42.322	5.146	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	586	848	0	0	0	0	0
N.S.	1	1.00	1.33	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	24.657	6.528	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	606	606	333	2250	0	0	0	0	0
N.S.	1	1.00	0.55	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	26.023	7.694	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1081	1080	10828	3389	0	0	0	0	0
N.S.	1	1.00	10.02	3.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.252	39.612	10.176	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	898	897	15131	1809	0	0	0	0	0
N.S.	1	1.00	16.85	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.717	35.437	5.174	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	443	1560	0	0	0	0	0
N.S.	1	1.00	0.94	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	36.156	5.172	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	723	855	0	0	0	0	0
N.S.	1	1.00	1.61	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	25.235	6.278	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	341	2298	0	0	0	0	0
N.S.	1	1.00	0.55	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	25.784	7.746	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1090	1090	10790	3571	0	0	0	0	0
N.S.	1	1.00	9.90	3.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.018	38.038	9.421	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	720	825	880	0	1267	0	0	0
N.S.	1	1.00	1.14	1.22	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	1.203	28.445	2.498	0.000	0.159	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	442	637	0	859	0	0	0
N.S.	1	1.00	1.08	1.55	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.397	25.003	2.336	0.000	0.123	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	326	506	0	682	0	0	0
N.S.	1	1.00	1.12	1.74	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.178	21.099	2.342	0.000	0.125	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	249	475	0	0	0	0	0
N.S.	1	1.00	0.81	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	22.056	3.028	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	3419	1211	0	0	0	0	0
N.S.	1	1.00	5.03	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.128	34.657	3.928	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	980	976	21961	1834	0	0	0	0	0
N.S.	1	1.00	22.41	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.893	36.913	5.277	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	734	732	8107	1552	0	0	0	0	0
N.S.	1	1.00	11.04	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	42.832	4.901	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	583	856	0	0	0	0	0
N.S.	1	1.00	1.34	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	25.181	6.313	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	616	616	340	2249	0	0	0	0	0
N.S.	1	1.00	0.55	3.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.813	26.043	7.511	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1128	1119	10836	3425	0	0	0	0	0
N.S.	1	0.99	9.61	3.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.987	40.007	10.267	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1097	1083	1291	1238	0	1665	0	0	0
N.S.	1	0.99	1.18	1.13	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	2.128	32.538	3.368	0.000	0.174	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	608	686	824	0	1068	0	0	0
N.S.	1	1.00	1.12	1.35	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.843	26.261	2.808	0.000	0.149	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	367	390	611	0	775	0	0	0
N.S.	1	1.00	1.06	1.66	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.321	23.770	2.322	0.000	0.146	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	1036	750	0	0	0	0	0
N.S.	1	1.00	2.23	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	26.549	3.018	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	738	3935	1269	0	0	0	0	0
N.S.	1	1.00	5.33	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.225	35.647	3.939	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1395	1376	39032	2228	0	0	0	0	0
N.S.	1	0.99	27.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.978	40.077	6.740	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	937	936	16972	1794	0	0	0	0	0
N.S.	1	1.00	18.11	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.556	36.207	5.232	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	757	757	6321	1065	0	0	0	0	0
N.S.	1	1.00	8.35	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	34.840	6.366	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	867	867	721	2286	0	0	0	0	0
N.S.	1	1.00	0.83	2.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.229	31.921	7.834	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1070	1070	11363	3342	0	0	0	0	0
N.S.	1	1.00	10.62	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.319	40.542	10.352	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [.2500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.00	40	0.200
2	A	8	7	1.00	38	0.184
3	A	6	5	1.00	33	0.152
4	A	9	7	1.00	40	0.175
5	A	12	10	1.00	40	0.250
6	A	10	10	0.99	42	0.238
7	A	7	7	1.00	42	0.167
8	A	5	5	1.00	42	0.119
9	A	7	7	1.00	42	0.167
10	A	8	7	1.00	42	0.167
11	A	10	10	1.00	49	0.204
12	A	5	5	1.00	49	0.102
13	A	5	5	1.00	49	0.102
14	A	7	7	1.00	49	0.143
15	A	8	7	1.00	49	0.143
16	A	8	7	1.00	58	0.121
17	A	7	6	1.00	53	0.113
18	A	7	6	1.00	60	0.100
19	A	10	8	1.00	60	0.133
20	A	13	11	1.00	60	0.183
21	A	9	9	1.00	62	0.145
22	A	8	8	1.00	62	0.129
23	A	6	6	1.00	62	0.097
24	A	8	8	1.00	62	0.129

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	9	8	0.99	62	0.129
26	A	9	8	0.99	42	0.190
27	A	8	7	1.00	40	0.175
28	A	7	6	1.00	35	0.171
29	A	11	9	1.00	42	0.214
30	A	12	10	1.00	42	0.238
31	A	10	10	0.99	44	0.227
32	A	9	9	1.00	44	0.204
33	A	8	8	1.00	44	0.182
34	A	9	9	1.00	44	0.204
35	A	8	8	1.00	44	0.182

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 700

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2b(7aBdfh + b(5Adfh - 4B(dfg + deh + cfh)))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15a^2Bd^2f^2h^2 + 10abdfh(3Adfh - 2B(dfg + deh + cfh)) - b^2(10Adfh(dfg + deh + cfh)}{15d^3f^{5/2}}$$
$$- \frac{2\sqrt{-de+cf}(15a^2d^2f^2h^2(Bg - Ah) + 10abdfh(3Adfgh - B(ch(fg - eh) + dg(2fg + eh))) - b^2(5Adf}{}$$

```
[Out] 2/15*b*(7*a*B*d*f*h+b*(5*A*d*f*h-4*B*(c*f*h+d*e*h+d*f*g)))*(d*x+c)^(1/2)*(f
*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b*B*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)
^(1/2)*(h*x+g)^(1/2)/d/f/h+2/15*(15*a^2*B*d^2*f^2*h^2+10*a*b*d*f*h*(3*A*d*f
*h-2*B*(c*f*h+d*e*h+d*f*g))-b^2*(10*A*d*f*h*(c*f*h+d*e*h+d*f*g)-B*(8*c^2*f^
2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*EllipticE(
f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c
*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(5/2)/h^3/(f
*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(15*a^2*d^2*f^2*h^2*(-A*h+B*g
)+10*a*b*d*f*h*(3*A*d*f*g*h-B*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))-b^2*(5*A*d*
f*h*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))-B*(4*c^2*f*h^2*(-e*h+f*g)+c*d*h*(-4*e^
2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*g*h+8*f^2*g^2))))*EllipticF
(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(
c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^
3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1611, 1614, 1629, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)(15a^2d^2f^2h^2(Bg-Ah)+10abdfh(}{}$$

$$+\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh}{15d^3f^{5/2}h}$$

$$+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(7aBdfh+5Abdfh-4bB(cfh+deh+dfg))}{15d^2f^2h^2}$$

$$+\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

```
[In] Int[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*(5*A*b*d*f*h + 7*a*B*d*f*h - 4*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(15*d^2*f^2*h^2) + (2*b*B*(a + b*x)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) + (2*Sqrt[-(d*e) + c*f]*(15*
a^2*B*d^2*f^2*h^2 + 10*a*b*d*f*h*(3*A*d*f*h - 2*B*(d*f*g + d*e*h + c*f*h))
- b^2*(10*A*d*f*h*(d*f*g + d*e*h + c*f*h) - B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f
*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f*x))/(
d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(15*d^3*f^(5/2)*h^3*Sqrt[e +
 f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^2*d^2*
f^2*h^2*(B*g - A*h) + 10*a*b*d*f*h*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g
*(2*f*g + e*h)) - b^2*(5*A*d*f*h*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) - B*
(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*
(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(15*d^3*f^(5/2)*h^3*Sqrt[e
 + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
```

```
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1611

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(
x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[
1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) + (A*b + a*B)*d*f*h*(2*m + 3)*x + b*
B*d*f*h*(2*m + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B},
x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(a+bx)\left(7aAdfh+7(Ab+aB)dfhx+7bBdfhx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{7dfh}$$

$$= \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{\int \frac{7dfh\left(5a^2Adfh-bB(2bceg+a(deg+cfg+ceh))\right)+7dfh(5a(2Ab+aB)dfh-bB(3b(deg+cfg+ceh)+2a(dfg+deh+cfh)))x+7bdfh(5A}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{35d^2f^2h^2}$$

$$= \frac{2b(5Abdfh+7aBdfh-4bB(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{2\int \frac{\frac{7}{2}d^2fh\left(15a^2Ad^2f^2h^2-10abBdfh(deg+cfg+ceh)-b^2\left(5Adfh(deg+cfg+ceh)-B\left(4d^2eg(fg+eh)+4c^2fh(fg+eh)+2cd\left(2f^2g^2+\right.\right.\right.\right.}{}}{}$$

$$= \frac{2b(5Abdfh + 7aBdfh - 4bB(dfg + deh + cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$- \frac{(15a^2d^2f^2h^2(Bg - Ah) + 10abdfh(3Adfgh - Bch(fg - eh) - Bdg(2fg + eh)) - b^2(5Adfh(ch}{}$$
$$+ \frac{(15a^2Bd^2f^2h^2 + 10abdfh(3Adfh - 2B(dfg + deh + cfh)) - b^2(10Adfh(dfg + deh + cfh) - B}{15d^2f^2h^3}$$

$$= \frac{2b(5Abdfh + 7aBdfh - 4bB(dfg + deh + cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$- \frac{\Big((15a^2d^2f^2h^2(Bg - Ah) + 10abdfh(3Adfgh - Bch(fg - eh) - Bdg(2fg + eh)) - b^2(5Adfh(c}{}$$
$$+ \frac{\Big((15a^2Bd^2f^2h^2 + 10abdfh(3Adfh - 2B(dfg + deh + cfh)) - b^2(10Adfh(dfg + deh + cfh) -}{15d^2f^2h}$$

$$= \frac{2b(5Abdfh + 7aBdfh - 4bB(dfg + deh + cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15a^2Bd^2f^2h^2 + 10abdfh(3Adfh - 2B(dfg + deh + cfh)) - b^2(10Adfh(dfg + deh}{1}$$
$$- \frac{\Big((15a^2d^2f^2h^2(Bg - Ah) + 10abdfh(3Adfgh - Bch(fg - eh) - Bdg(2fg + eh)) - b^2(5Adfh(c}{}$$

$$= \frac{2b(5Abdfh + 7aBdfh - 4bB(dfg + deh + cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15a^2Bd^2f^2h^2 + 10abdfh(3Adfh - 2B(dfg + deh + cfh)) - b^2(10Adfh(dfg + deh}{1}$$
$$- \frac{2\sqrt{-de+cf}(15a^2d^2f^2h^2(Bg - Ah) + 10abdfh(3Adfgh - Bch(fg - eh) - Bdg(2fg + eh)) -}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.41 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}(15a^2Bd^2f^2h^2-10abdfh(-3Adfh+2B(dfg+deh+cfh))+b^2(-10Adfh(dfg+deh\right.}{}$$

```
[In] Integrate[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
]),x]
```

```
[Out] (-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*a^2*B*d^2*f^2*h^2 - 10*a*b*d*f*h*(-3*A*d*
f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d*f*g + d*e*h + c*f*
h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h
+ 8*e^2*h^2))))*(e + f*x)*(g + h*x)) + b*d^2*Sqrt[-c + (d*e)/f]*f*h*(c + d*
x)*(e + f*x)*(g + h*x)*(-5*A*b*d*f*h - 10*a*B*d*f*h + b*B*(4*c*f*h + d*(4*f
*g + 4*e*h - 3*f*h*x))) - I*(d*e - c*f)*h*(15*a^2*B*d^2*f^2*h^2 - 10*a*b*d*
f*h*(-3*A*d*f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d*f*g +
d*e*h + c*f*h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2
+ 7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]
/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(15*a^2*d^2*f^2*(
-(B*e) + A*f)*h^2 + 10*a*b*d*f*h*(-3*A*d*e*f*h + B*c*f*(-(f*g) + e*h) + B*d
*e*(f*g + 2*e*h)) - b^2*(-5*A*d*f*h*(c*f*(-(f*g) + e*h) + d*e*(f*g + 2*e*h)
) + B*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*g*h + 3*e^2*h^2
) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e
 + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSi
nh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(1
5*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2B\,b^{2}x\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(b^{2}A+2abB-\frac{2B\,b^{2}(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh}}{3}\right.}{}$
default	Expression too large to display

```
[In] int((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/5*B*b^2/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*
d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*
g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*B*b^2/d/f/h*c*e*g-2/3*(b^2*A+2*a*b*B-2/5*B*b^
2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(
g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/
h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*
g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d)
)^(1/2))+2*(2*a*b*A+a^2*B-2/5*B*b^2/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2
/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(c*f*h+d
*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)
*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1
/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),
((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1236, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 2/45*(3*(3*B*b^2*d^3*f^3*h^3*x - 4*B*b^2*d^3*f^3*g*h^2 - (4*B*b^2*d^3*e*f^2
 + (4*B*b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt(f
*x + e)*sqrt(h*x + g) - (8*B*b^2*d^3*f^3*g^3 + (3*B*b^2*d^3*e*f^2 + (3*B*b^
2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*f^3)*g^2*h + (3*B*b^2*d^3*e^2*f + (3*B*
b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*e*f^2 + (3*B*b^2*c^2*d - 5*(2*B*a*b +
A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*f^3)*g*h^2 + (8*B*b^2*d^3*e^3 + (3
*B*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*e^2*f + (3*B*b^2*c^2*d - 5*(2*B*a*
b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*e*f^2 + (8*B*b^2*c^3 - 45*A*a^
2*d^3 - 10*(2*B*a*b + A*b^2)*c^2*d + 15*(B*a^2 + 2*A*a*b)*c*d^2)*f^3)*h^3)*
sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h
+ (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 -
3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)
*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*
f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) - 3*(8*B*b^2*d^3
*f^3*g^2*h + (7*B*b^2*d^3*e*f^2 + (7*B*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3
)*f^3)*g*h^2 + (8*B*b^2*d^3*e^2*f + (7*B*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d
^3)*e*f^2 + (8*B*b^2*c^2*d - 10*(2*B*a*b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a
*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2
*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*
f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d
^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^
3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))))/(
d^4*f^4*h^4)
```

Sympy [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**2*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x
)
```

```
[Out] Integral((A + B*x)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x))
, x)
```

Maxima [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

[In] integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)

Giac [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

[In] integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")

[Out] integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

[In] int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)

[Out] int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)

3.2 $\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 405

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$+\frac{2\sqrt{-de+cf}(3aBdfh+b(3Adfh-2B(dfg+deh+cfh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-c}{f(dg-}\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$-\frac{2\sqrt{-de+cf}(3adfh(Bg-Ah)+b(3Adfgh-B(ch(fg-eh)+dg(2fg+eh))))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{Ellip}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2/3*b*B*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*(3*a*B*d*f*h+b*
(3*A*d*f*h-2*B*(c*f*h+d*e*h+d*f*g)))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d
*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c
*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h
+d*g))^(1/2)-2/3*(3*a*d*f*h*(-A*h+B*g)+b*(3*A*d*f*g*h-B*(c*h*(-e*h+f*g)+d*g
*(e*h+2*f*g))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)
*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*
x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1611, 1629, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)(3adfh(Bg-Ah)+b(3Adfgh-}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$+\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+df}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$+\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

```
[In] Int[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*B*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + (2*Sqrt[-(d*e
) + c*f]*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[(
d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c +
d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)
*h^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h*(B*g - A*h) + b*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g*(2*f*g
 + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*E
llipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h
)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1611

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(
x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[
1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) + (A*b + a*B)*d*f*h*(2*m + 3)*x + b*
B*d*f*h*(2*m + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B},
x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{5aAdfh+5(Ab+aB)dfhx+5bBdfhx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{5dfh}$$

$$= \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{2\int \frac{\frac{5}{2}d^2fh(3aAdfh-bB(deg+cfg+ceh))+\frac{5}{2}d^2fh(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{15d^3f^2h^2}$$

$$= \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3dfh^2} - \frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3dfh^2}$$

$$= \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{\left((3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}\cdots}}}{3dfh^2\sqrt{e+fx}} + \frac{\left((3Abdfh+3aBdfh-2bB(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{3dfh^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{2\sqrt{-de+cf}(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\cdots\right.\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{\left((3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{\cdots}{\sqrt{c+\cdots}}}{3dfh^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{2\sqrt{-de+cf}(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\cdots\right.\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g\cdots}{dg\cdots}}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.19 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2bBd^2fh(e+fx)(g+hx) - \frac{2d^2(-3Abdfh-3aBdfh+2bB(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx} + \frac{2i(de-cf)h(3Abdfh+3aB\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] (Sqrt[c + d*x]*(2*b*B*d^2*f*h*(e + f*x)*(g + h*x) - (2*d^2*(-3*A*b*d*f*h -
3*a*B*d*f*h + 2*b*B*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*x)
 + ((2*I)*(d*e - c*f)*h*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*h +
 c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))
/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f
*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f] + ((2*I)*d*h*(3*a*d*f*(-(B
*e) + A*f)*h + b*(-3*A*d*e*f*h + B*c*f*(-(f*g) + e*h) + B*d*e*(f*g + 2*e*h)
))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2Bb\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(Aa-\frac{2Bb\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\cdots}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/3*B*b/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*
g*x+c*e*g)^(1/2)+2*(A*a-2/3*B*b/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h
-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e
/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x
+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(
1/2))+2*(A*b+B*a-2/3*B*b/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h
-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x
^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/
h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c
/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 842, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}Bbd^2f^2h^2+(2\,Bbd^2f^2g^2+(Bbd^2ef+(Bbcd-3\,(Ba+Ab)d^2)f^2)gh+(2\right.}{}$$

```
[In] integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] 2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*B*b*d^2*f^2*h^2 + (2*B*b*d
^2*f^2*g^2 + (B*b*d^2*e*f + (B*b*c*d - 3*(B*a + A*b)*d^2)*f^2)*g*h + (2*B*b
*d^2*e^2 + (B*b*c*d - 3*(B*a + A*b)*d^2)*e*f + (2*B*b*c^2 + 9*A*a*d^2 - 3*(
B*a + A*b)*c*d)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2
- (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2
), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f -
4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f
^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)
/(d*f*h)) + 3*(2*B*b*d^2*f^2*g*h + (2*B*b*d^2*e*f + (2*B*b*c*d - 3*(B*a + A
*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2
*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*
f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d
^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
```

```
 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^
3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))))/(
d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((A + B*x)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x
)
```

Maxima [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Giac [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
[Out] integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))
,x)
```

```
[Out] int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))
, x)
```

3.3 $\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*B*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-A*h+B*g)*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

= {164, 115, 114, 122, 121}

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2(Bg-Ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Int[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(B*g - A*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
```

```
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{h}+\frac{(-Bg+Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}\\
&= \frac{\left((-Bg+Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{h\sqrt{e+fx}}\\
&\quad+\frac{\left(B\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&= \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&\quad+\frac{\left((-Bg+Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{h\sqrt{e+fx}\sqrt{g+hx}}\\
&= \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&\quad-\frac{2\sqrt{-de+cf}(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-Bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-iB(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{d}{f}}}{\sqrt{c+dx}}\right.\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c+}}$$

[In] Integrate[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]

[Out] (-2*(-(B*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*B*(d*e - c*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*(B*e - A*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2A\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2B\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\left(\right.}{\sqrt{dfh\,x^3+cfh\,x^2+}}\right.}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}$
default	$-\frac{2\left(AF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)de\,h^2-AF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)dfgh-BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ce\,h^2+BF\left(\sqrt{-}\right.\right.}{}$

[In] int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(2*A*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h

```
+c/d))^(1/2))+2*B*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^
(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f
))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(
1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(3\sqrt{dfh}Bdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3d^2f^2h^2},-\frac{4\left(2d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\left(d\right.\right.}{}\right.\right.}{}$$

```
[In] integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(3*sqrt(d*f*h)*B*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c
*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^
3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3
)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d
*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*
f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 +
c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*
h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))) + (B*
d*f*g + (B*d*e + (B*c - 3*A*d)*f)*h)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d
^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(
d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*
e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((A + B*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
[Out] integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
[Out] integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.4 $\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 313

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A-\frac{aB}{b}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*B*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d
*g))^(1/2)/b/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A-a*B/b)*EllipticPi(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)
*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*
x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

= {1621, 175, 552, 551, 12, 122, 121}

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}$$

```
[In] Int[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/
(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
 - (2*(A - (a*B)/b)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\ &\quad +\int \frac{B}{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\ &= \frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b}-\Bigg(2\bigg(A \\ &\quad -\frac{aB}{b}\bigg)\Bigg)\operatorname{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx,x,\sqrt{c+dx}\right)\end{aligned}$$

$$= \frac{\left(B\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b\sqrt{e+fx}}$$

$$- \frac{\left(2\left(A-\frac{aB}{b}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}}$$

$$= \frac{\left(B\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$- \frac{\left(2\left(A-\frac{aB}{b}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$- \frac{2\left(A-\frac{aB}{b}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.95 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$= \frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(b(-Bc+Ad)\operatorname{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right), \frac{dfg-cfh}{deh-cfh}\right)+(-Ab+aB)d\operatorname{EllipticPi}\left(\right.\right.}{b(-bc+ad)\sqrt{-c+\frac{de}{f}}f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

```
[In] Integrate[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] ((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(b*(-(B*c) + A*d)*El
lipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h
 - c*f*h)] + (-(A*b) + a*B)*d*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f))
, I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f
*h)]))/(b*(-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f*x))/(f*(c + d*
x))]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2B\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2(Ab-Ba)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\cdots}}}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+ceh\cdots}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\left(A\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-gb)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)be\,h^2-A\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(e\cdots}{f(\cdots}\right.\right.}{\cdots}$

[In] int((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(2*B/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*(A*b-B*a)/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

[In] integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((A + B*x)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x
)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
[Out] integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
[Out] integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x
)
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),
x)
```

3.5 $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 678

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$+\frac{\sqrt{-de+cf}(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(d}{b(bc-ad)^2\sqrt{f}(be-af)(bg-ah)}$$

```
[Out] -b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e
)/(-a*h+b*g)/(b*x+a)+(A*b-B*a)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1
/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e)/(
-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)^(1/
2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+(3*a^2*A*b*d*f*h-a^3*B*d*f*h-b^3*(2*B*c*e*g
-A*(c*e*h+c*f*g+d*e*g))+a*b^2*(B*(c*e*h+c*f*g+d*e*g)-2*A*(c*f*h+d*e*h+d*f*g
)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*
c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*
e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)
/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-(A*b-B*a)*EllipticF(f^(1/2)*(d*x+c)^(1
/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1
/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b/(-a*d+b*c)/
(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1613, 1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2(b}$$
$$-\frac{\sqrt{f}(Ab-aB)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{e+fx}\sqrt{g+hx}(bc-ad)(be-af)}$$
$$+\frac{\sqrt{f}\sqrt{g+hx}(Ab-aB)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{e+fx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

```
[In] Int[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] -((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b
*e - a*f)*(b*g - a*h)*(a + b*x))) + ((A*b - a*B)*Sqrt[f]*Sqrt[-(d*e) + c*f]
*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c -
 a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]
) - ((A*b - a*B)*Sqrt[f]*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]
*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*(b*c - a*d)*(b*e -
 a*f)*Sqrt[e + f*x]*Sqrt[g + h*x]) + (Sqrt[-(d*e) + c*f]*(3*a^2*A*b*d*f*h -
 a^3*B*d*f*h - b^3*(2*B*c*e*g - A*(d*e*g + c*f*g + c*e*h)) + a*b^2*(B*(d*e*
g + c*f*g + c*e*h) - 2*A*(d*f*g + d*e*h + c*f*h)))*Sqrt[(d*(e + f*x))/(d*e
- c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c
- a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/(b*(b*c - a*d)^2*Sqrt[f]*(b*e - a*f)*(b*g - a*h)*Sqr
t[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
```

```
- a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\int \frac{-2a^2Adfh+b^2(2Bceg-A(deg+cfg+ceh))-ab(B(deg+cfg+ceh)-2A(dfg+deh+cfh))+2a(Ab-aB)dfhx+b(Ab-aB)dfhx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\int \frac{aAdfh-\frac{a^2Bdfh}{b}+(Abdfh-aBdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2(bc-ad)(be-af)(bg-ah)}$$

$$- \frac{(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(dfg+}{2b(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$- \frac{((Ab-aB)df)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b(bc-ad)(be-af)} + \frac{((Ab-aB)df)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2(bc-ad)(be-af)(bg-ah)}$$

$$+ \frac{(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(dfg+}{b(bc-ad)(be-af)(bg-a}$$

$$= -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$- \frac{\left((Ab-aB)df\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{2b(bc-ad)(be-af)\sqrt{e+fx}}$$

$$+ \frac{\left((3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(dfg\right.}{b(bc-ad)(be-af)(bg-}$$

$$+ \frac{\left((Ab-aB)df\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{2(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$+ \frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$- \frac{\left((Ab-aB)df\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{2b(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$

$$+ \frac{\left((3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(dfg\right.}{b(bc-ad)(be-af)(bg-}$$

$$= -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$+\frac{\sqrt{-de+cf}(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh}{b(bc-ad)^2\sqrt{f}(be-af)(bg-}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.13 (sec) , antiderivative size = 3412, normalized size of antiderivative = 5.03

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] -((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b
*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(A*b^3*c*Sqrt[-c + (d*
e)/f]*f*h - a*b^2*B*c*Sqrt[-c + (d*e)/f]*f*h - a*A*b^2*d*Sqrt[-c + (d*e)/f]
*f*h + a^2*b*B*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^3*c*d^2*e*Sqrt[-c + (d*e)/f]
*g)/(c + d*x)^2 - (a*b^2*B*c*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (a*A
*b^2*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (a^2*b*B*d^3*e*Sqrt[-c + (d*
e)/f]*g)/(c + d*x)^2 - (A*b^3*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (
a*b^2*B*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^2*c*d^2*Sqrt[-c
+ (d*e)/f]*f*g)/(c + d*x)^2 - (a^2*b*B*c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d
*x)^2 - (A*b^3*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2*B*c^2*d*e
*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^2*c*d^2*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x)^2 - (a^2*b*B*c*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (A*b^3*
c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^3*Sqrt[-c + (d*e)/f]*f
*h)/(c + d*x)^2 - (a*A*b^2*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (a^2
*b*B*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (A*b^3*c*d*Sqrt[-c + (d*e)
/f]*f*g)/(c + d*x) - (a*b^2*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a*A*
b^2*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b*B*d^2*Sqrt[-c + (d*e)/f]
*f*g)/(c + d*x) + (A*b^3*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*b^2*B*c
*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*A*b^2*d^2*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x) + (a^2*b*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (2*A*b^3*c^2*
Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (2*a*b^2*B*c^2*Sqrt[-c + (d*e)/f]*f*h)/
```

```
(c + d*x) + (2*a*A*b^2*c*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) - (2*a^2*b*B*c
*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (I*b*(A*b - a*B)*(-(b*c) + a*d)*(-(d
*e) + c*f)*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*
x) + (d*g)/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d
*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + (I*b*d*(2*b*B*c*e -
 A*b*(d*e + c*f) - a*(B*d*e + B*c*f - 2*A*d*f))*(-(b*g) + a*h)*Sqrt[1 - c/(
c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]
*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*
e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*b^3*B*c*d*e*g*Sqrt[1 - c/(c + d*x) +
(d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi
[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*A*b^3*d^2*e*g*S
qrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*
(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x
] - (I*a*b^2*B*d^2*e*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 -
 c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b
*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h
 - c*f*h)])/Sqrt[c + d*x] - (I*A*b^3*c*d*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(
f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c
*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
(d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*a*b^2*B*c*d*f*g*Sqrt[1
 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c +
d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (
d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + (
(2*I)*a*A*b^2*d^2*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 -
c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*
c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h
- c*f*h)])/Sqrt[c + d*x] - (I*A*b^3*c*d*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f
*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*
f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (
d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*a*b^2*B*c*d*e*h*Sqrt[1
- c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d
*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d
*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((
2*I)*a*A*b^2*d^2*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c
/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c
*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h -
 c*f*h)])/Sqrt[c + d*x] + ((2*I)*a*A*b^2*c*d*f*h*Sqrt[1 - c/(c + d*x) + (d*
e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-(
(b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x
]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((3*I)*a^2*A*b*d^2*f*
h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/
(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqr
t[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c +
```

```
d*x] + (I*a^3*B*d^2*f*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1
- c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e -
b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*
h - c*f*h)])/Sqrt[c + d*x]))/(b*d*(b*c - a*d)*(-(b*c) + a*d)*Sqrt[-c + (d*e
)/f]*(-(b*e) + a*f)*(-(b*g) + a*h)*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)
))/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1208
default	Expression too large to display	13344

```
[In] int((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*
b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e
*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(A*b-B*a)/(a^3*d*f*h-a^2*
b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e
*g)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g
*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g
/h+c/d))^(1/2))-d*f*h*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(g/h-e/f)*((x+g/h)/(g/h-
e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^
3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h
+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/
d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))+(3*A*
a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*b^3*c*e*h+A*b
^3*c*f*g+A*b^3*d*e*g-B*a^3*d*f*h+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-
2*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a
*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*(
(x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d
*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*Elliptic
Pi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^
(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x
)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2))
,x)
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2))
, x)
```

3.6 $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 981

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(5aBdfh+b(4Adfh-3B(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$
$$+\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(5aBdfh+b(4Adfh-3B(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-}}{\sqrt{fg-}}\right.\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(3aBdfh+b(4Adfh-B(cfh+3d(fg+eh))))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\text{arcsin}\right.}{4bdf^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+\frac{\sqrt{-dg+ch}(4dfh(2a(2Ab+aB)dfh-bB(b(deg+cfg+ceh)+a(dfg+deh+cfh)))-(adfh+b(dfg+}{}$$

```
[Out] 1/4*(4*d*f*h*(2*a*(2*A*b+B*a)*d*f*h-b*B*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e
*h+d*f*g)))-(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(5*a*B*d*f*h+b*(4*A*d*f*h-3*B*(
c*f*h+d*e*h+d*f*g))))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*
h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g
)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d
*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/d^2/f^2/
h^3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*(5*a*B*d*f*h+b*(4*A*d*
f*h-3*B*(c*f*h+d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f
^2/h^2/(d*x+c)^(1/2)+1/2*b*B*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x
+g)^(1/2)/d/f/h-1/4*(-a*f+b*e)*(3*a*B*d*f*h+b*(4*A*d*f*h-B*(c*f*h+3*d*(e*h+
f*g))))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(
```

```
1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)
*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b/d/f^2/h^2/(-
e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)
-1/4*(5*a*B*d*f*h+b*(4*A*d*f*h-3*B*(c*f*h+d*e*h+d*f*g)))*EllipticE((-c*h+d*
g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g
)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(
1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d^2/f^2/h^2/((-c*f+d*e)
*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 976, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1611, 1614, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg}}{\sqrt{fg}}\right.\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(4Abdfh+3aBdfh-bB(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\text{arcsi}\right.}{4bdf^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$-\frac{\sqrt{ch-dg}((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-4dfh(2a(2Ab+a}{}$$
$$+\frac{(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$

```
[In] Int[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] ((4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*
Sqrt[e + f*x]*Sqrt[g + h*x])/(4*d*f^2*h^2*Sqrt[c + d*x]) + (b*B*Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) - (Sqrt[d*g - c*h]*
Sqrt[f*g - e*h]*(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))
*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Ell
ipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x
])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(4*d^2*f^2*h^2*S
qrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((b*e
 - a*f)*Sqrt[b*g - a*h]*(4*A*b*d*f*h + 3*a*B*d*f*h - b*B*(c*f*h + 3*d*(f*g
+ e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
+ b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(4*b*d*f
```

```
^2*h^2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
 e*h)*(a + b*x)))]) - (Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*
f*h))*(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h)) - 4*d*f*h
*(2*a*(2*A*b + a*B)*d*f*h - b*B*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e
*h + c*f*h))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x
))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(
d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[
-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g
- e*h))])/(4*b*d^2*Sqrt[b*c - a*d]*f^2*h^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1611

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(
x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[
1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) + (A*b + a*B)*d*f*h*(2*m + 3)*x + b*
B*d*f*h*(2*m + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B},
x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+bx}\left(6aAdfh+6(Ab+aB)dfhx+6bBdfhx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{6dfh}$$

$$= \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{6dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+12dfh\left(2a(2Ab+aB)dfh-bB(b(deg+cfg+ceh)+a(dfg+deh+cfh))\right)x+6bdfh(4Abd\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{24d^2f^2h^2}$$

$$= \frac{(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} + \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{-6bdfh\left((bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)\right)-6bdfh((adfh+b(\cdots}{\sqrt{a+bx}\sqrt{c+dx}\cdots}}{48bd^3\cdots} + \frac{((de-cf)(dg-ch)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8d^2f^2h^2}$$

$$= \frac{(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} + \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} - \frac{((be-af)(bg-ah)(4Abdfh+3aBdfh-bB(cfh+3d(fg+eh))))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8bdf^2h^2} - \frac{((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-4dfh(2a(2Ab+a\cdots}{8bd^2f^2h^2} - \frac{\left((dg-ch)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-\cdots}}{\sqrt{1\cdots}}\cdots\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$
\begin{aligned}
&= \frac{(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} \\
&+ \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} \\
&- \frac{\sqrt{dg-ch}\sqrt{fg-eh}(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\Big(\mathrm{s}}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} \\
&- \frac{\Big(((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-4dfh(2a(2Ab+}{} \\
&- \frac{\Big((be-af)(bg-ah)(4Abdfh+3aBdfh-bB(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\Big)\,\mathrm{Su}}{4bdf^2h^2(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}} \\
&= \frac{(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} \\
&+ \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} \\
&- \frac{\sqrt{dg-ch}\sqrt{fg-eh}(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\Big(\mathrm{s}}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} \\
&- \frac{(be-af)\sqrt{bg-ah}(4Abdfh+3aBdfh-bB(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\Big(\mathrm{sin}}{4bdf^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} \\
&- \frac{\sqrt{-dg+ch}((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-4df\mathrm{h}}{}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. 2(981) = 1962.

Time = 36.59 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.39

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1813 vs. 2(898) = 1796.

Time = 5.18 (sec) , antiderivative size = 1814, normalized size of antiderivative = 1.85

method	result	size
elliptic	Expression too large to display	1814
default	Expression too large to display	55936

```
[In] int((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(1/2*B*b/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*
d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f
*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2
*(a^2*A-1/2*B*b/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g
/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)
*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^
(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)
*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b*A+a^2*B-1/2*B*b/d/f/h*(a*c*f*h+a*
d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h
+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2
)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*
f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a
/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(
1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-
g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(b^
2*A+2*a*b*B-1/2*B*b/d/f/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g)
)*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d)
)^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)
*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h
+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((
e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/
h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-
c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h
+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-
c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a
/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(A+Bx)\,(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

```
[Out] Integral((A + B*x)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^
(1/2)),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^
(1/2)), x)
```

3.7 $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 736

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{B\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{B(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\sqrt{-dg+ch}(2Abdfh+B(adfh-b(dfg+deh+cfh)))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\operatorname{EllipticPi}}{bd\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] (2*A*b*d*f*h+B*(a*d*f*h-b*(c*f*h+d*e*h+d*f*g)))*(b*x+a)*EllipticPi((-a*d+b*
c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*
c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*(
(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)
/(b*x+a))^(1/2)/b/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+B*(b
*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h/(d*x+c)^(1/2)-B*(-a*f+b*e)*Elli
pticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d
+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((-a*f+b*e)
*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b/f/h/(-e*h+f*g)^(1/2)/(d*
x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-B*EllipticE((-c*h
+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+
f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a
)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d/f/h/((-c*f+d*e)*(b
*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1610, 176, 430, 182, 435, 171, 551}

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(aBdfh+2Abdfh-bB(cfh+deh+dfg))\,\text{EllipticPi}\left(-\frac{b(c}{(bc}\right.}{bdfh^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$
$$-\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$
$$-\frac{B\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}$$
$$+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

```
[In] Int[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] (B*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (B*Sqrt
[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((
f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g
 - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqr
t[g + h*x]) - (B*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(b*f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b
*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[-(d*g) + c*h]*(2*A*
b*d*f*h + a*B*d*f*h - b*B*(d*f*g + d*e*h + c*f*h))*(a + b*x)*Sqrt[((b*g - a
*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g
- e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(S
qrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e -
a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*d*Sqrt[b*c - a*d]*f*h^2*Sq
rt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
```

```
 - e*h)*(a + b*x))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1610

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Dis
t[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[B*(d*e - c*f)*((d*g - c*h)/(2*
d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x
], x] + Dist[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b*d
*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e + c*
f), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} \\
&\quad + \frac{1}{2}\left(2A+B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)\right)\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&\quad - \frac{(B(be-af)(bg-ah))\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bfh} \\
&\quad + \frac{(B(de-cf)(dg-ch))\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2dfh} \\
&= \frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} \\
&\quad + \frac{\left(\left(2A+B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)\right)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int\frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}}\right.}{\sqrt{c+dx}\sqrt{e+fx}} \\
&\quad - \frac{\left(B(be-af)(bg-ah)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{bfh(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}} \\
&\quad - \frac{\left(B(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}
\end{aligned}
$$

$$= \frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{B\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{B(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\left(2A+B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)\right)\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-}\right.}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8030 vs. 2(736) = 1472.

Time = 42.32 (sec) , antiderivative size = 8030, normalized size of antiderivative = 10.91

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. 2(671) = 1342.

Time = 5.15 (sec) , antiderivative size = 1544, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	21369

```
[In] int((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*A*a*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b
)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*
(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(A*b+B*a)*
```

```
(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/
b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d)
)^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)
*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+B*b*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g
/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e
/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/
b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x
+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))
^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*
f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/
(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

```
[Out] Integral((A + B*x)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^
(1/2)),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^
(1/2)), x)
```

3.8 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 442

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab-aB)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{2B\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)}{(bc-ad)}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 2*B*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+
a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h
+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)
*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/h/(-a*d+b*c)^(1/2)/(d*x+c)
^(1/2)/(f*x+e)^(1/2)+2*(A*b-B*a)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(
-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g)
)^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b/(-a*
h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)
/(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {1612, 176, 430, 171, 551}

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$
$$+ \frac{2B(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(a}{(bc-ad)(f}\right.}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g
 + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*S
qrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b
*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x
))/((f*g - e*h)*(a + b*x)))]) + (2*B*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*
g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/(
(f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcS
in[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b
*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c - a*d]*h*Sqr
t[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
h}, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b}$$

$$= \frac{\left(2B(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g+hx}}{\sqrt{a+bx}}\right)}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$+\frac{\left(2(Ab-aB)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{b(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2(Ab-aB)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+\frac{2B\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-af)(dg}{(bc-ad)(fg}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [A] (verified)

Time = 24.66 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{Ab\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)(a+bx)\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}-\frac{aB\sqrt{\frac{(bg-a}{(fg-e}}}{}\right.}{}$$

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
]),x]
```

```
[Out] (2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-
((A*b*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ellip
ticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(
b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a + b
*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (a*B*Sqrt[
((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*EllipticF[ArcSin
[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)
*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a + b*x)*Sqrt
[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (B*(-(f*g) + e*h)*S
qrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x
)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[Sqrt[((-(b*e)
+ a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h)
)/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(b*Sqrt[c + d*x]*Sqrt[e + f
*x]*Sqrt[g + h*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(404) = 808.

Time = 6.53 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2A\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}},\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.}{}$
default	Expression too large to display

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*A*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))
^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*
(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x
+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*B*(g/h-a/b)*
((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)
/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-
g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*E
llipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g
/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1
/2)),x)
```

```
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1
/2)), x)
```

3.9 $\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 606

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{2(Bc-Ad)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[Out] 2*(A*b-B*a)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*
e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+
g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*(-A*d+B*c)*Ellipt
icF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b
*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)
/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/
(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*(A*b-B*a)*El
lipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*
d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)
^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/(-a*d+b
*c)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*
x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx = \frac{2\sqrt{g+hx}(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}}{\sqrt{fg-eh}}\right.\right.}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+h}{(a+b}}}$$

$$-\frac{2\sqrt{a+bx}(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{\sqrt{g+hx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{c+dx}(bc-ad)(be-af)(bg-ah)}$$

```
[In] Int[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] (2*(A*b - a*B)*d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b
*e - a*f)*(b*g - a*h)*Sqrt[c + d*x]) - (2*b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x])
 - (2*(A*b - a*B)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*
e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g -
c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e
*h))/((b*e - a*f)*(d*g - c*h))])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[
((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + (2*(B*c -
 A*d)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*E
llipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b
*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d
)*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*
x))/((f*g - e*h)*(a + b*x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
```

```
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad+\frac{\int \frac{b^2Bceg-a^2Adfh-ab(B(deg+cfg+ceh)-A(dfg+deh+cfh))+(Ab-aB)(adfh+b(dfg+deh+cfh))x+2b(Ab-aB)dfhx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(be-af)(bg-ah)} \\
&= \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} \\
&\quad-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}+\frac{\int \frac{2bd(Bc-Ad)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bd(bc-ad)f(be-af)h(bg-ah)} \\
&\quad+\frac{((Ab-aB)(de-cf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(be-af)(bg-ah)} \\
&= \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad+\frac{(Bc-Ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} \\
&\quad-\frac{\left(2(Ab-aB)(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} \\
&= \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad-\frac{2(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} \\
&\quad+\frac{\left(2(Bc-Ad)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}
\end{aligned}
$$

$$= \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{2(Bc-Ad)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] (verified)

Time = 26.02 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((Ab-aB)(dg\right.}{(bc}$$

```
[In] Integrate[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x]),x]
```

```
[Out] (2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e + f
*x)^(3/2)*(g + h*x)^(3/2)*((A*b - a*B)*(d*g - c*h)*EllipticE[ArcSin[Sqrt[((
-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h
))/((b*e - a*f)*(d*g - c*h))] + (B*c - A*d)*(b*g - a*h)*EllipticF[ArcSin[Sq
rt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g
- e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*g - e*h)^3*(a + b*x)^(
5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g -
 e*h)^2*(a + b*x)^2)))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2249 vs. 2(552) = 1104.

Time = 7.69 (sec) , antiderivative size = 2250, normalized size of antiderivative = 3.71

method	result	size
elliptic	Expression too large to display	2250
default	Expression too large to display	18724

```
[In] int((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*
```

```
c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2
*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)/((x+a/b)*
(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*
g*x+b*c*e*g))^(1/2)+2*(B/b+1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2
*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^
2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b*c*f*g+b
*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*
f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a
/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*
((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*
h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*
(-(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*
d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h
+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*
c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(g/h-a/b)*((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(
x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/
d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g
/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(
-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))
^(1/2)))-2*b*d*f*h*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g
+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*((x+a/b)*(x+e/f)*(x+g/h)+(g
/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)
*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^
(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)
/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(
a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*
(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^
(1/2))
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/
(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*h
)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a*b*
c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g + (a^
2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c + a^
2*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Bx}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1
/2)),x)
```

```
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1
/2)), x)
```

3.10 $\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 1081

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg \cdots}{\cdots}$$

$$-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2b(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))-a^2 \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg \cdots}{3(bc-ad)^2(be-af)^2 \cdots}$$

$$-\frac{2(3a^2d(Bc-Ad)fh+b^2(3Bcdeg-A(2d^2eg-c^2fh+cd(fg+eh)))+ab(3Ad^2(fg+eh)-B(d^2eg+c^2f \cdots}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg- \cdots}}$$

```
[Out] 2/3*d*(3*a^3*B*d*f*h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c*e*
h+c*f*g+d*e*g)-4*A*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+d*f
*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(
-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+
g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*b*(3*a^3*B*d*f*
h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c*e*h+c*f*g+d*e*g)-4*A*
(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+d*f*g)))*(d*x+c)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)
^(1/2)-2/3*(3*a^2*d*(-A*d+B*c)*f*h+b^2*(3*B*c*d*e*g-A*(2*d^2*e*g-c^2*f*h+c*
d*(e*h+f*g)))+a*b*(3*A*d^2*(e*h+f*g)-B*(d^2*e*g+c^2*f*h+2*c*d*(e*h+f*g))))*
EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(
-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*
```

```
f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(3/2
)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(
1/2)-2/3*(3*a^3*B*d*f*h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c
*e*h+c*f*g+d*e*g)-4*A*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+
d*f*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^
(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)
*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1
/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(
d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 1080, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg}{3(bc-ad)^2(be-af}$$

$$-\frac{2(3d(Bc-Ad)fha^2+b(3Ad^2(fg+eh)-B(fhc^2+2d(fg+eh)c+d^2eg))\,a+b^2(Afhc^2+3Bdegc-A}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg}}$$

$$-\frac{2b(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$+\frac{2d(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

```
[In] Int[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] (2*d*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h)) - a*b^2
*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*A*d*f
*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[c + d*x]) - (2*b*(A*b -
a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)*(a + b*x)^(3/2)) - (2*b*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*(
d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g + d
*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^
2*Sqrt[a + b*x]) - (2*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(3*a^3*B*d*f*h + b^3*
(3*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h
```

```
) - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*
f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))
)]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c
 + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(3*(b*c -
a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f
)*(c + d*x))]*Sqrt[g + h*x]) - (2*(3*a^2*d*(B*c - A*d)*f*h + b^2*(3*B*c*d*e
*g - 2*A*d^2*e*g + A*c^2*f*h - A*c*d*(f*g + e*h)) + a*b*(3*A*d^2*(f*g + e*h
) - B*(d^2*e*g + c^2*f*h + 2*c*d*(f*g + e*h))))*Sqrt[((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*
Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h)
)/((d*e - c*f)*(b*g - a*h)))])/(3*(b*c - a*d)^2*(b*e - a*f)*(b*g - a*h)^(3/
2)*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = -\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{\int \frac{-3a^2Adfh+b^2(3Bceg-2A(deg+cfg+ceh))-ab(B(deg+cfg+ceh)-3A(dfg+deh+cfh))+(Ab-aB)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} - \frac{2b(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg+deh-}{3(bc-ad)^2(be-af)^2(bg-ah)} + \frac{\int \frac{b(Ab-aB)(bceg-a(deg+cfg+ceh))(3adfh-b(dfg+deh+cfh))+a(adfh-b(dfg+deh+cfh))(3a^2Adfh-b^2(3Bceg-2A(deg+cfg}{}}{}$$

$$= \frac{2d(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + cf \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(Ab - aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2 \cdots}$$
$$+ \frac{\int -\frac{2bdf(be-af)h(bg-ah)\left(3a^2d(Bc-Ad)fh+b^2\left(3Bcdeg-2Ad^2eg+Ac^2fh-Acd(fg+eh)\right)+ab\left(3Ad^2(fg+eh)-B\left(d^2eg+c^2fh+2cd \cdots\right.\right.\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6bd(bc-ad)^2f(be-af)^2h(bg-ah)^2}$$
$$+ \frac{((de-cf)(dg-ch)\left(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - \cdots\right.}{3(bc-ad)^2(be-af \cdots}$$

$$= \frac{2d(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + cf \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(Ab - aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2 \cdots}$$
$$- \frac{(3a^2d(Bc - Ad)fh + b^2(3Bcdeg - 2Ad^2eg + Ac^2fh - Acd(fg + eh)) + ab(3Ad^2(fg + eh) - B(\cdots}{3(bc-ad)^2(be-af)(bg-ah)}$$
$$- \frac{\left(2(dg - ch)\left(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(df \cdots\right.\right.}{3(bc-ad)^2(be- \cdots}$$

$$= \frac{2d(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + cf \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(Ab - aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2 \cdots}$$
$$- \frac{2\sqrt{dg-ch}\sqrt{fg-eh}(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) \cdots}{3(bc-ad)^2(\cdots}$$
$$- \frac{\left(2(3a^2d(Bc - Ad)fh + b^2(3Bcdeg - 2Ad^2eg + Ac^2fh - Acd(fg + eh)) + ab(3Ad^2(fg + eh) - \cdots\right.}{3(bc-ad)^2(be-af)(bg- \cdots}$$

$$= \frac{2d(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh + \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{\cdots}}$$
$$- \frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh) - 4A(dfg + deh - \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)\cdots}$$
$$- \frac{2\sqrt{dg-ch}\sqrt{fg-eh}(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh)) - ab^2(B(deg + cfg + ceh)\cdots}{3(bc-ad)\cdots}$$
$$- \frac{2(3a^2d(Bc-Ad)fh + b^2(3Bcdeg - 2Ad^2eg + Ac^2fh - Acd(fg+eh)) + ab(3Ad^2(fg+eh) - \cdots}{3(bc-ad)^2(be-af)(bg-ah)^{3/\cdots}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10828 vs. 2(1081) = 2162.

Time = 39.61 (sec) , antiderivative size = 10828, normalized size of antiderivative = 10.02

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. 2(1009) = 2018.

Time = 10.18 (sec) , antiderivative size = 3389, normalized size of antiderivative = 3.14

method	result	size
elliptic	Expression too large to display	3389
default	Expression too large to display	104801

```
[In] int((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2/3/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)*(b*d*f*h*x^4+a*d*f
```

```
*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*
x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e
*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*
f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b
*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(6*A*a^
2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A
*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*h+B*a^2*b*
d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g)/((x+a/b)*(b*
d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x
+b*c*e*g))^(1/2)+2*(-1/3*(3*A*a*b*d*f*h-A*b^2*c*f*h-A*b^2*d*e*h-A*b^2*d*f*g
-3*B*a^2*d*f*h+B*a*b*c*f*h+B*a*b*d*e*h+B*a*b*d*f*g)/b/(a^3*d*f*h-a^2*b*c*f*
h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)+1/
3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)
*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c
*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*h+
B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g)/(a^3
*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g)^2-1/3*(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b
*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(6*A*a^
2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A
*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*h+B*a^2*b*
d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g))*(g/h-a/b)*(
(-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/
(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g
/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*Elliptic
F(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+
e/f)/(-c/d+g/h))^(1/2))+2*(-1/3*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(6*A*a^2*
b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A*b
^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*h+B*a^2*b*d*
f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g)/(a^3*d*f*h-a^2
*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*
e*g)^2-1/3*(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e
*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(6*A*a^2*b*
d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A*b^3
*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*h+B*a^2*b*d*f*
g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g))*(g/h-a/b)*((-g/
h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/
f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c
/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*Ellipt
icF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/
b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/
b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c
/d+g/h))^(1/2)))-2/3*b*d*f*h*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e
*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+
B*a^2*b*c*f*h+B*a^2*b*d*e*h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b
```

```
^2*d*e*g-3*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^
2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-
a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x
+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/
2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/
h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-
c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d
*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+
a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-
a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2))
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/
(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d)*f)
*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*e + 3
*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c + a^2
*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^3 + ((3
*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a^2*b*c +
 a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*g)*x), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1
/2)),x)
```

```
[Out] int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1
/2)), x)
```

3.11 $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 898

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}}$$
$$+\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(3adfh+b(cfh-d(3fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+f}}{\sqrt{fg-eh}\sqrt{a+b}}\right.\right.}{2bfh^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$-\frac{\sqrt{-dg+ch}\left(6abd^2f^2gh-3a^2d^2f^2h^2+b^2(2cdefh^2-c^2f^2h^2-d^2(3f^2g^2+e^2h^2))\right)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}{2bd\sqrt{bc-ad}fh^3\sqrt{c+dx}\sqrt{e+f}}$$

```
[Out] -1/2*(6*a*b*d^2*f^2*g*h-3*a^2*d^2*f^2*h^2+b^2*(2*c*d*e*f*h^2-c^2*f^2*h^2-d^
2*(e^2*h^2+3*f^2*g^2)))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(
c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d
*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h
+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/d/f/h^
3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/2*(5*a*d*f*h-b*(c*f*h+d*e*
h+3*d*f*g))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h^2/(d*x+c)^(1/2)+b
*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h-1/2*(-a*f+b*e)*(
3*a*d*f*h+b*(c*f*h-d*(e*h+3*f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)
/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*
g))^(1/2))*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(
h*x+g)^(1/2)/b/f/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-
```

```
e*h+f*g)/(b*x+a))^(1/2)-1/2*(5*a*d*f*h-b*(c*f*h+d*e*h+3*d*f*g))*EllipticE((
-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-
e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b
*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d/f/h^2/((-c*f+d
*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {1611, 1614, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-a}{(be-a}\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+\frac{(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(bcfh+3adfh-bd(3fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)\right.}{2fh^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}b}$$
$$-\frac{\sqrt{ch-dg}\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}{2d\sqrt{bc-ad}fh^3\sqrt{c+dx}\sqrt{e+fx}}$$

```
[In] Int[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x]),x]
```

```
[Out] ((5*a*d*f*h - b*(3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/(2*f*h^2*Sqrt[c + d*x]) + (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x])/h - (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(5*a*d*f*h - b*(
3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*
g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqr
t[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g -
 c*h))])/(2*d*f*h^2*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*S
qrt[g + h*x]) - ((b*e - a*f)*Sqrt[b*g - a*h]*(b*c*f*h + 3*a*d*f*h - b*d*(3*
f*g + e*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g +
h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt
[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(2*b*
f*h^2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]) - (Sqrt[-(d*g) + c*h]*(6*a*b*d^2*f^2*g*h - 3*a^2*d^2*f^2
*h^2 + b^2*(2*c*d*e*f*h^2 - c^2*f^2*h^2 - d^2*(3*f^2*g^2 + e^2*h^2)))*(a +
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
```

```
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(2*b*d*Sqr
t[b*c - a*d]*f*h^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1611

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(
x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[
1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) + (A*b + a*B)*d*f*h*(2*m + 3)*x + b*
B*d*f*h*(2*m + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B},
x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
```

```
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+bx}\left(6adf(de+cf)h+6df(bde+bcf+2adf)hx+12bd^2f^2hx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{6dfh}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h} + \frac{\int \frac{12d^2f^2h\left(2a^2(de+cf)h-b(bceg+a(deg+cfg+ceh))\right)+24d^2f^2h\left(2a^2dfh-b^2(deg+cfg+ceh)-ab(dfg-deh-cfh)\right)x+12bd^2f^2h(5a\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{24d^2f^2h^2}$$

$$= \frac{(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}} + \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h} + \frac{\int \frac{12bd^2f^2h\left(a^2df(4de-cf)h^2+b^2deg(3dfg+deh-cfh)-abfh\left(7d^2eg-c^2fh-cd(fg-eh)\right)\right)-12bd^2f^2h\left(6abd^2f^2gh-3a^2d^2f^2h^2+b^2\cdots\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{48bd^3f^3h^3} + \frac{((de-cf)(dg-ch)(5adfh-b(3dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{4dfh^2}$$

$$= \frac{(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}} + \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h} - \frac{1}{4}\left(-\frac{3a^2df}{b} + b\left(2ce-\frac{de^2}{f}-\frac{c^2f}{d}-\frac{3dfg^2}{h^2}\right)+\frac{6adfg}{h}\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx - \frac{((be-af)(bg-ah)(bcfh+3adfh-bd(3fg+eh)))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{4bfh^2} - \frac{\left((dg-ch)(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx,\cdots\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{(5adfh - b(3dfg + deh + cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}} + \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h} - \frac{\sqrt{dg-ch}\sqrt{fg-eh}(5adfh - b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+}}{\sqrt{fg-eh}\sqrt{c+}}\right.\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{\left(\left(-\frac{3a^2df}{b} + b\left(2ce - \frac{de^2}{f} - \frac{c^2f}{d} - \frac{3dfg^2}{h^2}\right) + \frac{6adfg}{h}\right)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int\right.}{2\sqrt{c+dx}\sqrt{e+fx}} - \frac{\left((be-af)(bg-ah)(bcfh+3adfh-bd(3fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+\frac{(bc-ad}{de-c}}}\right.}{2bfh^2(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{(5adfh - b(3dfg + deh + cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}} + \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h} - \frac{\sqrt{dg-ch}\sqrt{fg-eh}(5adfh - b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+}}{\sqrt{fg-eh}\sqrt{c+}}\right.\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{(be-af)\sqrt{bg-ah}(bcfh+3adfh-bd(3fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right.\right.}{2bfh^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\left(\frac{3a^2df}{b} - b\left(2ce - \frac{de^2}{f} - \frac{c^2f}{d} - \frac{3dfg^2}{h^2}\right) - \frac{6adfg}{h}\right)\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(\right.}{2\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 15131 vs. $2(898) = 1796$.

Time = 35.44 (sec) , antiderivative size = 15131, normalized size of antiderivative = 16.85

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f
*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. 2(817) = 1634.

Time = 5.17 (sec) , antiderivative size = 1809, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1809
default	Expression too large to display	35482

```
[In] int((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(b/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+
b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d
*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*c*f+
a^2*d*e-b/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g/h-a/b)*((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(
-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/
h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF
(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e
/f)/(-c/d+g/h))^(1/2))+2*(2*a^2*d*f+2*a*c*f*b+2*a*b*d*e-b/h*(a*c*f*h+a*d*e*
h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((
-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*
(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)
)+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+
a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(4*a*d*
f*b+b^2*c*f+b^2*d*e-b/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g))*
((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^
(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(
x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c
/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/
f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/
d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c
/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/
d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b
)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h
*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*x
)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(c\,f+d\,e+2\,d\,f\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2
)*(c + d*x)^(1/2)),x)
```

```
[Out] int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2
)*(c + d*x)^(1/2)), x)
```

3.12 $\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 472

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2d(bg-ah)^{3/2}\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),\frac{(de-cf)(bg}{(be-af)(dg}\right.}{\sqrt{be-af}\,h^2\sqrt{a+bx}\sqrt{c+dx}}$$

```
[Out] -2*d*(-a*h+b*g)^(3/2)*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-a
*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e)*(-a*h+b*g
)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*((-e*h+f*g)*(b*x+a)/(-a*h+b*g)/(f*x+e))^(1/
2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2)/h^2/(-a*f+b*e)^(1/2)/(b*x+
a)^(1/2)/(d*x+c)^(1/2)+2*b*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h/(b*x
+a)^(1/2)-2*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+
a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(
1/2)*(-e*h+f*g)^(1/2)*(d*x+c)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a)
)^(1/2)/h/((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {1609, 171, 551, 182, 435}

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2d(e+fx)(bg-ah)^{3/2}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),\frac{(de-cf)(}{(be-af)(}\right.}{h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$

$$-\frac{2\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

$$+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

```
[In] Int[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]),x]
```

```
[Out] (2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(h*Sqrt[a + b*x]) - (2*Sqrt
[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((
f*g - e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b
*g - a*h)))])/(h*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt
[g + h*x]) - (2*d*(b*g - a*h)^(3/2)*Sqrt[((f*g - e*h)*(a + b*x))/((b*g - a*
h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x))]*(e + f
*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e - a*f]*Sqr
t[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g - a*h))/((b
*e - a*f)*(d*g - c*h))])/(Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
```

```
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1609

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Dist[B*((b*g - a*h)/(2*f*h)), Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[g + h*x]), x], x] + Dist[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h)), Int
[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Rubi steps

$$\text{integral} = \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{(d(bg-ah))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}\,dx}{h} + \frac{((be-af)(bg-ah))\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}$$

$$= \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{\left(2d(bg-ah)\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\right)\text{Subst}\left(\int\frac{1}{(h-fx^2)\sqrt{1+\frac{(-be+af)x^2}{bg-ah}}\sqrt{1+\frac{(-de+cf)x^2}{dg-ch}}}\right.}{h\sqrt{a+bx}\sqrt{c+dx}} - \frac{\left(2(bg-ah)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}}{\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{h\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$= \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2d(bg-ah)^{3/2}\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\Pi\left(\frac{f(bg-ah)}{(be-af)h};\sin^{-1}\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(de-cf)(bg-}{(be-af)(dg-}\right.}{\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}}$$

Mathematica [A] (verified)

Time = 36.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.94

$$\int\frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx = -\frac{2\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{dh(e+fx)(g+hx)}{c+dx} - \frac{(fg-eh)\sqrt{\frac{(-de+cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}\left((de-cf)hE\left(\arcsin\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-}{(de-}\right.\right.}{}\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x
]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[c + d*x]*(-((d*h*(e + f*x)*(g + h*x))/(c + d*x)) - (
(f*g - e*h)*Sqrt[((-(d*e) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e
*h)^2*(c + d*x)^2)]*((d*e - c*f)*h*EllipticE[ArcSin[Sqrt[((-(d*e) + c*f)*(g
 + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*
f)*(b*g - a*h))] + (-(d*e*h) + c*f*h)*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)
*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e -
 c*f)*(b*g - a*h))] + f*(d*g - c*h)*EllipticPi[(d*(-(f*g) + e*h))/((d*e - c
*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], (
(b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))]))/((d*e - c*f)*Sqrt[
((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])))/(h^2*Sqrt[e + f*x]*Sqrt
[g + h*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(426) = 852$.

Time = 5.17 (sec) , antiderivative size = 1560, normalized size of antiderivative = 3.31

method	result	size
elliptic	Expression too large to display	1560
default	Expression too large to display	13180

```
[In] int((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(a*c*f+a*d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a
/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*
((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*
h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*
(2*a*d*f+b*c*f+b*d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/
2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g
/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d
)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*El
lipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d
),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+2*b*d*f*((x+a/b)*(x+e
/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)
^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+
a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b
)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a
/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)
)/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticP
i(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-
c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+
e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h
*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*x)*s
qrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*
sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*
sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(c\,f+d\,e+2\,d\,f\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2
)*(c + d*x)^(1/2)),x)
```

```
[Out] int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2
)*(c + d*x)^(1/2)), x)
```

3.13 $\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 449

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bde+bcf-2adf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{4df\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-\cdots}{(bc-\cdots}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 4*d*f*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*
x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e
*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/
2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/h/(-a*d+b*c)^(1/2)/(d*x+
c)^(1/2)/(f*x+e)^(1/2)+2*(-2*a*d*f+b*c*f+b*d*e)*EllipticF((-a*h+b*g)^(1/2)*
(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+
d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+
g)^(1/2)/b/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*
x+g)/(-e*h+f*g)/(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {1612, 176, 430, 171, 551}

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$
$$+\frac{4df(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg}{(bc-ad)(fg}\right.}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

```
[In] Int[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]),x]
```

```
[Out] (2*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
 b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqr
t[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g
 - a*h)))])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e -
 a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (4*d*f*Sqrt[-(d*g) + c*h]*(a
+ b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a
*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c
- a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt
[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[
b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
```

```
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rubi steps

$$\text{integral} = \frac{(2df)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(-2adf+b(de+cf))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b}$$

$$= \frac{\left(4df(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g+hx}}{\sqrt{a+bx}}\right)}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$+ \frac{\left(2(-2adf+b(de+cf))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right.}{b(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2(bde+bcf-2adf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+ \frac{4df\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-af)(d}{(bc-ad)(f}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [A] (verified)

Time = 25.23 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.61

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-bde(be-af)h\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc}{(be}\right.\right.}{}$$

```
[In] Integrate[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-(b
*d*e*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g
 + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a +
b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) + 2*a*
d*f*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g
+ h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b
*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] + b*c*f*
(-(b*e) + a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g +
 h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*
x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] - 2*d*f*(
b*g - a*h)*(f*g - e*h)*(a + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x))]*Sqrt[((-(b*e) + a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g
- e*h)^2*(a + b*x)^2)]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSi
n[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d
)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/(b*(b*e - a*f)*h*(b*g - a*h)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/
((f*g - e*h)*(a + b*x))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(411) = 822.

Time = 6.28 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2(cf+de)\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)(x}{\left(-\frac{g}{h}+\frac{a}{b}\right)(x}}\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.}{}$
default	Expression too large to display

```
[In] int((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(c*f+d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c
/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x
+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/
b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+4*d*f*
(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/
b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d)
)^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)
*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{cf+de+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h
*x+g)**(1/2),x)
```

```
[Out] Integral((c*f + d*e + 2*d*f*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*s
qrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*
sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*
sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{c\,f+d\,e+2\,d\,f\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*
(c + d*x)^(1/2)),x)
```

```
[Out] int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*
(c + d*x)^(1/2)), x)
```

3.14 $\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 625

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2(bde+bcf-2adf)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{2d(de-cf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[Out] 2*d*(-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+
b*c)/(-a*f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b*(-2*a*d*f+b*c*f+b*d*e)*(d*x+c)
^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)
^(1/2)-2*d*(-c*f+d*e)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(
1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+
b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(
b*x+a))^(1/2)-2*(-2*a*d*f+b*c*f+b*d*e)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(
1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h
+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*
(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d
*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2d\sqrt{g+hx}(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} - \frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}(-2adf+bcf+bde)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{\sqrt{g+hx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}} - \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)} + \frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{c+dx}(bc-ad)(be-af)(bg-ah)}$$

```
[In] Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]),x]
```

```
[Out] (2*d*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(
(b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[c + d*x]) - (2*b*(b*d*e + b*c*f -
2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f
)*(b*g - a*h)*Sqrt[a + b*x]) - (2*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[d*g - c*h]
*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*
(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e
*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/
((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a
*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*d*(d*e - c*f)*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
```

```
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
```

```
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad+\frac{\int \frac{2b^2cdefg-a^2df(de+cf)h-ab(cdf^2g-c^2f^2h+d^2e(fg-eh))+(bde+bcf-2adf)(adfh+b(dfg+deh+cfh))x+2bdf(bde+bcf-2adf)hx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(be-af)(bg-ah)} \\
&= \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} \\
&\quad-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad+\frac{\int -\frac{2bd^2f(be-af)(de-cf)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bd(bc-ad)f(be-af)h(bg-ah)} \\
&\quad+\frac{((de-cf)(bde+bcf-2adf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(be-af)(bg-ah)} \\
&= \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} \\
&\quad-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} \\
&\quad-\frac{(d(de-cf))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} \\
&\quad-\frac{\left(2(bde+bcf-2adf)(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}
\end{aligned}$$

$$= \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(bde+bcf-2adf)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-a}{(be-a}\right.}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{\left(2d(de-cf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(bde+bcf-2adf)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-a}{(be-a}\right.}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{2d(de-cf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] (verified)

Time = 25.78 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.55

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((bde+bcf-\right.}{}$$

```
[In] Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x
]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e + f
*x)^(3/2)*(g + h*x)^(3/2)*((b*d*e + b*c*f - 2*a*d*f)*(d*g - c*h)*EllipticE[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a
*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))] - d*(d*e - c*f)*(b*g - a*h)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((
b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*g - e*
h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g
 + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(571) = 1142$.

Time = 7.75 (sec) , antiderivative size = 2298, normalized size of antiderivative = 3.68

method	result	size
elliptic	Expression too large to display	2298
default	Expression too large to display	21256

```
[In] int((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(-2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b
*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^
2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d
*e)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c
*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(2/b*d*f-1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*
e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(2*a*d*f-b*c*f-b*d*e)/(a^3*d*f
*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-
b^3*c*e*g)+(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2
*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*
e))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/
d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+
c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))
^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(
g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*((a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)
*(2*a*d*f-b*c*f-b*d*e)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2
*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)+(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(
a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2
*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*e))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/
2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d
*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+
a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^
(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(
-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+2*
b*d*f*h*(2*a*d*f-b*c*f-b*d*e)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*
g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*((x+a/b)*(x+e/f)*(x+g/h)+(
g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b
)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))
^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF((
(-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x
+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+
```

```
(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)
*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b
)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))
^(1/2))
```

Fricas [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{3}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sq
rt(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2
*a*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)
*e + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d
)*f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (
2*a*b*c + a^2*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{cf + de + 2dfx}{(a + bx)^{\frac{3}{2}}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h
*x+g)**(1/2),x)
```

```
[Out] Integral((c*f + d*e + 2*d*f*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x
)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{3}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{3}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{c\,f + d\,e + 2\,d\,f\,x}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,(a + b\,x)^{3/2}\,\sqrt{c + d\,x}}\,dx$$

```
[In] int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*
(c + d*x)^(1/2)),x)
```

```
[Out] int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*
(c + d*x)^(1/2)), x)
```

3.15 $\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 1090

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{4d(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(f\ldots}{\ldots}$$
$$-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$-\frac{4b(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))+ab^2(2c^2f^2h+d^2\ldots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}\ldots}$$
$$-\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))\ldots}{3(bc-ad)^2(be-af)^2(\ldots}$$
$$+\frac{2(de-cf)\left(3a^2d^2fh-abd(dfg+3deh+2cfh)+b^2(2d^2eg-cdfg+cdeh+c^2fh)\right)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+h\ldots}}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-a\ldots}{(fg-e\ldots}}}$$

```
[Out] 4/3*d*(3*a^3*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-c*d
*e*(-e*h+f*g)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d*f*(
3*e*h+f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b
*e)^2/(-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b*(-2*a*d*f+b*c*f+b*d*e)*(d*x+c)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)
-4/3*b*(3*a^3*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-c*
d*e*(-e*h+f*g)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d*f*
(3*e*h+f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+
b*e)^2/(-a*h+b*g)^2/(b*x+a)^(1/2)+2/3*(-c*f+d*e)*(3*a^2*d^2*f*h-a*b*d*(2*c*
f*h+3*d*e*h+d*f*g)+b^2*(c^2*f*h+c*d*e*h-c*d*f*g+2*d^2*e*g))*EllipticF((-a*h
+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h
+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))
```

```
^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(3/2)/(-e*h+f*g)^(1
/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-4/3*(3*a^3
*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-c*d*e*(-e*h+f*g
)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d*f*(3*e*h+f*g)))
*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((
-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f
*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/(-a*
d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^
(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bde+bcf-2adf)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2 \cdots}{3(bc-ad)^2(be-af)^2(b \cdots}$$

$$+\frac{2(de-cf)\left(\left(fhc^2-dfgc+dehc+2d^2eg\right)b^2-ad(dfg+3deh+2cfh)b+3a^2d^2fh\right)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx \cdots}}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)}{(fg-eh)} \cdots}}$$

$$-\frac{4b(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h)a-b^3(f(fg+e \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$+\frac{4d(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h)a-b^3(f(fg+e \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

```
[In] Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]),x]
```

```
[Out] (4*d*(3*a^3*d^2*f^2*h - a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^
2*g - c*d*e*(f*g - e*h) + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(
f*g + 2*e*h) + c*d*f*(f*g + 3*e*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[c + d*x]) - (2*b*(b*
d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c -
 a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - (4*b*(3*a^3*d^2*f^2*h - a^
2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*(f*g - e*h) +
c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h) + c*d*f*(f*g
+ 3*e*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)^2*(b*e
```

```
 - a*f)^2*(b*g - a*h)^2*Sqrt[a + b*x]) - (4*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]
*(3*a^3*d^2*f^2*h - a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g
- c*d*e*(f*g - e*h) + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g
+ 2*e*h) + c*d*f*(f*g + 3*e*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x
))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x
])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)
*(d*g - c*h))])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[((d*e - c
*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + (2*(d*e - c*f)*(3*
a^2*d^2*f*h - a*b*d*(d*f*g + 3*d*e*h + 2*c*f*h) + b^2*(2*d^2*e*g - c*d*f*g
+ c*d*e*h + c^2*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]
*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g -
 e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)
))])/(3*(b*c - a*d)^2*(b*e - a*f)*(b*g - a*h)^(3/2)*Sqrt[f*g - e*h]*Sqrt[c
+ d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = -\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{\int \frac{2bdf(3bceg-a(deg+cfg+ceh))-(de+cf)\left(3a^2dfh+2b^2(deg+cfg+ceh)-3ab(dfg+deh+cfh)\right)+(bde+bcf-2adf)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} - \frac{4b(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))+ab^2(2c^2 \cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a \cdots}} + \frac{\int \frac{b(bde+bcf-2adf)(bceg-a(deg+cfg+ceh))(3adfh-b(dfg+deh+cfh))-a(adfh-b(dfg+deh+cfh))\left(2bdf(3bceg-a(deg+cfg+ceh) \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{4d(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c^2 \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(bde + bcf - 2adf)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{3(bc - ad)(be - af)(bg - ah)(a + bx)^{3/2}}$$
$$- \frac{4b(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{\cdots}}$$
$$+ \frac{\int \frac{2bdf(be-af)(de-cf)h(bg-ah)\left(3a^2d^2fh-abd(dfg+3deh+2cfh)+b^2\left(2d^2eg-cdfg+cdeh+c^2fh\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{6bd(bc - ad)^2f(be - af)^2h(bg - ah)^2}$$
$$+ \frac{(2(de - cf)(dg - ch)\left(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f \cdots\right.}{3(bc - ad)^2(be - af \cdots}$$

$$= \frac{4d(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c^2 \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(bde + bcf - 2adf)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{3(bc - ad)(be - af)(bg - ah)(a + bx)^{3/2}}$$
$$- \frac{4b(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{\cdots}}$$
$$+ \frac{((de - cf)\left(3a^2d^2fh - abd(dfg + 3deh + 2cfh) + b^2(2d^2eg - cdfg + cdeh + c^2fh)\right)) \int \frac{\cdots}{\sqrt{a+bx}\sqrt{c \cdots}}}{3(bc - ad)^2(be - af)(bg - ah)}$$
$$- \frac{\left(4(dg - ch)\left(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh \cdots\right.\right.}{3(bc - ad)^2(be - \cdots}$$

$$= \frac{4d(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c^2 \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{c \cdots}}$$
$$- \frac{2b(bde + bcf - 2adf)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{3(bc - ad)(be - af)(bg - ah)(a + bx)^{3/2}}$$
$$- \frac{4b(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c \cdots}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{\cdots}}$$
$$- \frac{4\sqrt{dg - ch}\sqrt{fg - eh}(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(\cdots}{3(bc - ad)^2(\cdots}$$
$$+ \frac{\left(2(de - cf)\left(3a^2d^2fh - abd(dfg + 3deh + 2cfh) + b^2(2d^2eg - cdfg + cdeh + c^2fh)\right)\sqrt{\frac{(be-af)(\cdots}{(de-cf)(\cdots}} \cdots\right.}{3(bc - ad)^2(be - af)(bg - ah)(fg - eh)\sqrt{c + d \cdots}}$$

$$= \frac{4d(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c^2f^2}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{c +}}$$

$$- \frac{2b(bde + bcf - 2adf)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{3(bc - ad)(be - af)(bg - ah)(a + bx)^{3/2}}$$

$$- \frac{4b(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(fg + eh)) + ab^2(2c^2}{3(bc - ad)^2(be - af)^2(bg - ah)^2\sqrt{a}}$$

$$- \frac{4\sqrt{dg - ch}\sqrt{fg - eh}(3a^3d^2f^2h - a^2bdf(dfg + 4deh + 4cfh) - b^3(d^2e^2g - cde(fg - eh) + c^2f(f}{3(bc - ad)^2(be}$$

$$+ \frac{2(de - cf)\left(3a^2d^2fh - abd(dfg + 3deh + 2cfh) + b^2(2d^2eg - cdfg + cdeh + c^2fh)\right)\sqrt{\frac{(be-af)(c+d}{(de-cf)(a+b}}}{3(bc - ad)^2(be - af)(bg - ah)^{3/2}\sqrt{fg - eh}\sqrt{c + dx}\sqrt{-}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10790 vs. 2(1090) = 2180.

Time = 38.04 (sec) , antiderivative size = 10790, normalized size of antiderivative = 9.90

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x
]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3570 vs. 2(1018) = 2036.

Time = 9.42 (sec) , antiderivative size = 3571, normalized size of antiderivative = 3.28

method	result	size
elliptic	Expression too large to display	3571
default	Expression too large to display	87910

```
[In] int((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(-2/3/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g
+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*e)*(b*d*
```

```
f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h
*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*
d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2-4/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d
*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*
b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e
*g)^2*(3*a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+
2*a*b^2*c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2
*d^2*e*f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*
e^2*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+
b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(1/3/b*(6*a^2*d^2*f^2*h-5*a*b*c*d*f^2
*h-5*a*b*d^2*e*f*h-2*a*b*d^2*f^2*g+b^2*c^2*f^2*h+2*b^2*c*d*e*f*h+b^2*c*d*f^
2*g+b^2*d^2*e^2*h+b^2*d^2*e*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d
*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-2/3/b*(a^2*d*f*h-a*b*c*
f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(3*a^3*d^2*f^2*h-4*a
^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2*
c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-b
^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2*g)/(a^3*d*f*h-a^2*b*c*
f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^
2+2/3*(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(3*a^3*d^2*f^2*h-4*a^2
*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2*c*
d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-b^3
*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2*g))*(g/h-a/b)*((-g/h+c/d
)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b
)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(
-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/
d+g/h))^(1/2))+2*(2/3*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(3*a^3*d^2*f^2*h-4*
a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2
*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-
b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2*g)/(a^3*d*f*h-a^2*b*c
*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)
^2+2/3*(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(3*a^3*d^2*f^2*
h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a
*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*
f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2*g))*(g/h-a/b)*((-
g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-
e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h
+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*Elli
pticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-
a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+
a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(
-c/d+g/h))^(1/2)))+4/3*b*d*f*h*(3*a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d
^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*
```

```
g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2
*h+b^3*c*d*e*f*g-b^3*d^2*e^2*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*((x+a/b)*(x+e/f)*(x+g/
h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c
/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*Ellipti
cF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b
+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a
/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h
-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g
/h))^(1/2))
```

Fricas [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sq
rt(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3
*a*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*
b^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a
*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*
h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f +
(3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)
*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h
*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
[Out] integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
[Out] integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

```
[In] int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*
(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.16 $\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 721

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2b^2(5bBdfh+2C(adfh-2b(dfg+deh+cfh)))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$- \frac{2b\sqrt{-de+cf}(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(dfg+deh+cfh))+b^2(10Bdfh(dfg+deh+cfh)}{15d^3f^{5/2}h}$$
$$- \frac{2\sqrt{-de+cf}(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Cg+Bh)+5ab^2dfh(6Bdfgh-C(ch(fg-eh)+dg(2fg+}{}$$

```
[Out] 2/15*b^2*(5*b*B*d*f*h+2*C*(a*d*f*h-2*b*(c*f*h+d*e*h+d*f*g)))*(d*x+c)^(1/2)*
(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b^2*C*(b*x+a)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-2/15*b*(15*a^2*C*d^2*f^2*h^2-10*a*b*d*f*h*(3
*B*d*f*h-C*(c*f*h+d*e*h+d*f*g))+b^2*(10*B*d*f*h*(c*f*h+d*e*h+d*f*g)-C*(8*c^
2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*Ellipt
icE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2)
)*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(5/2)/h^
3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(15*a^3*C*d^2*f^2*h^3-15*
a^2*b*d^2*f^2*h^2*(B*h+C*g)+5*a*b^2*d*f*h*(6*B*d*f*g*h-C*(c*h*(-e*h+f*g)+d*
g*(e*h+2*f*g)))-b^3*(5*B*d*f*h*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))-C*(4*c^2*f*
h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*
g*h+8*f^2*g^2))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*
```

```
e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(
h*x+g)/(-c*h+d*g))^(1/2)/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1614, 1629, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cf\right.}{15d^3f^{5/2}}$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh\right.}{}$$

$$+\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(2aCdfh+5bBdfh-4bC(cfh+deh+dfg))}{15d^2f^2h^2}$$

$$+\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

```
[In] Int[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*Sqrt[e
 + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b^2*(5*b*B*d*f*h + 2*a*C*d*f*h - 4*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(15*d^2*f^2*h^2) + (2*b^2*C*(a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) - (2*b*Sqrt[-(d*e) + c*f
]*(15*a^2*C*d^2*f^2*h^2 - 10*a*b*d*f*h*(3*B*d*f*h - C*(d*f*g + d*e*h + c*f*
h)) + b^2*(10*B*d*f*h*(d*f*g + d*e*h + c*f*h) - C*(8*c^2*f^2*h^2 + 7*c*d*f*
h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f*x
))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt
[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(15*d^3*f^(5/2)*h^3*Sqrt
[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^3*
C*d^2*f^2*h^3 - 15*a^2*b*d^2*f^2*h^2*(C*g + B*h) + 5*a*b^2*d*f*h*(6*B*d*f*g
*h - c*C*h*(f*g - e*h) - C*d*g*(2*f*g + e*h)) - b^3*(5*B*d*f*h*(c*h*(f*g -
e*h) + d*g*(2*f*g + e*h)) - C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 +
 e*f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(
d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[
(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h)
)])/(15*d^3*f^(5/2)*h^3*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
```

```
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
```

```
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{\int \frac{5a^2(bB-aC)dfh-b^2C(2bceg+a(deg+cfg+ceh))+b(5a(2bB-aC)dfh-bC(3b(deg+cfg+ceh)+2a(dfg+deh+cfh)))x+b^2(5bBdfh+2aCdf\cdots}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5dfh}$$

$$= \frac{2b^2(5bBdfh+2aCdfh-4bC(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{2\int \frac{1}{2}d(15a^2bBd^2f^2h^2-15a^3Cd^2f^2h^2-5ab^2Cdfh(deg+cfg+ceh)-b^3(5Bdfh(deg+cfg+ceh)-C(4d^2eg(fg+eh)+4c^2fh(fg+eh\cdots}{\cdots}$$

$$= \frac{2b^2(5bBdfh+2aCdfh-4bC(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} - \frac{(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Cg+Bh)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(2fg+eh\cdots}{\cdots} - \frac{(b(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(dfg+deh+cfh))+b^2(10Bdfh(dfg+deh+cfh)-\cdots}{15d^2f^2h^3}$$

$$= \frac{2b^2(5bBdfh + 2aCdfh - 4bC(dfg + deh + cfh))\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{15d^2f^2h^2}$$
$$+ \frac{2b^2C(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{5dfh}$$
$$- \frac{\Big((15a^3Cd^2f^2h^3 - 15a^2bd^2f^2h^2(Cg + Bh) + 5ab^2dfh(6Bdfgh - cCh(fg - eh) - Cdg(2fg + eh}{}$$
$$- \frac{\Big(b(15a^2Cd^2f^2h^2 - 10abdfh(3Bdfh - C(dfg + deh + cfh)) + b^2(10Bdfh(dfg + deh + cfh) - C}{15d^2f^2h^3}$$

$$= \frac{2b^2(5bBdfh + 2aCdfh - 4bC(dfg + deh + cfh))\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{15d^2f^2h^2}$$
$$+ \frac{2b^2C(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{5dfh}$$
$$- \frac{2b\sqrt{-de + cf}(15a^2Cd^2f^2h^2 - 10abdfh(3Bdfh - C(dfg + deh + cfh)) + b^2(10Bdfh(dfg + deh}{15}$$
$$- \frac{\Big((15a^3Cd^2f^2h^3 - 15a^2bd^2f^2h^2(Cg + Bh) + 5ab^2dfh(6Bdfgh - cCh(fg - eh) - Cdg(2fg + eh}{}$$

$$= \frac{2b^2(5bBdfh + 2aCdfh - 4bC(dfg + deh + cfh))\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{15d^2f^2h^2}$$
$$+ \frac{2b^2C(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{5dfh}$$
$$- \frac{2b\sqrt{-de + cf}(15a^2Cd^2f^2h^2 - 10abdfh(3Bdfh - C(dfg + deh + cfh)) + b^2(10Bdfh(dfg + deh}{15}$$
$$- \frac{2\sqrt{-de + cf}(15a^3Cd^2f^2h^3 - 15a^2bd^2f^2h^2(Cg + Bh) + 5ab^2dfh(6Bdfgh - cCh(fg - eh) - Cd}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.44 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(bd^2\sqrt{-c+\frac{de}{f}}(15a^2Cd^2f^2h^2+10abdfh(-3Bdfh+C(dfg+deh+cfh))-b^2(-10Bdfh(dfg+deh\right.}{}$$

```
[In] Integrate[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(b*d^2*Sqrt[-c + (d*e)/f]*(15*a^2*C*d^2*f^2*h^2 + 10*a*b*d*f*h*(-3*B*d*
f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f*g + d*e*h + c*f*h)
 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h +
8*e^2*h^2))))*(e + f*x)*(g + h*x) + b^2*d^2*Sqrt[-c + (d*e)/f]*f*h*(c + d*x
)*(e + f*x)*(g + h*x)*(-5*b*B*d*f*h - 5*a*C*d*f*h + b*C*(4*c*f*h + d*(4*f*g
 + 4*e*h - 3*f*h*x))) + I*b*(d*e - c*f)*h*(15*a^2*C*d^2*f^2*h^2 + 10*a*b*d*
f*h*(-3*B*d*f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f*g + d*
e*h + c*f*h) + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 +
7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]
*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/S
qrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(15*a^3*C*d^2*f^3*h
^2 - 15*a^2*b*d^2*f^2*(C*e + B*f)*h^2 - 5*a*b^2*d*f*h*(-6*B*d*e*f*h + c*C*f
*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) + b^3*(-5*B*d*f*h*(c*f*(-(f*g) + e*h
) + d*e*(f*g + 2*e*h)) + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2
+ e*f*g*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c +
d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x
))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/
(d*e*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\,b^3x\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(B\,b^3+C\,b^2a-\frac{2C\,b^3(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh\,x \cdots}}{3d \cdots}\right.}{\cdots}$
default	Expression too large to display

[In] int((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(2/5*C*b^3/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)+2/3*(B*b^3+C*b^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)+2*(a^2*b*B-C*a^3-2/5*C*b^3/d/f/h*c*e*g-2/3*(B*b^3+C*b^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*(2*a*b^2*B-C*a^2*b-2/5*C*b^3/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2/3*(B*b^3+C*b^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1267, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

[In] integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")

[Out] 2/45*(3*(3*C*b^3*d^3*f^3*h^3*x - 4*C*b^3*d^3*f^3*g*h^2 - (4*C*b^3*d^3*e*f^2 + (4*C*b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt(f

```
*x + e)*sqrt(h*x + g) - (8*C*b^3*d^3*f^3*g^3 + (3*C*b^3*d^3*e*f^2 + (3*C*b^
3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g^2*h + (3*C*b^3*d^3*e^2*f + (3*C*
b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (3*C*b^3*c^2*d - 5*(C*a*b^2 +
B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^3*e^3
+ (3*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*e^2*f + (3*C*b^3*c^2*d - 5*(C*
a*b^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*e*f^2 + (8*C*b^3*c^3 -
 10*(C*a*b^2 + B*b^3)*c^2*d - 15*(C*a^2*b - 2*B*a*b^2)*c*d^2 + 45*(C*a^3 -
B*a^2*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 -
(d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2),
 -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*
c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2
 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(
d*f*h)) - 3*(8*C*b^3*d^3*f^3*g^2*h + (7*C*b^3*d^3*e*f^2 + (7*C*b^3*c*d^2 -
10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^3*e^2*f + (7*C*b^3*c*d^2
- 10*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (8*C*b^3*c^2*d - 10*(C*a*b^2 + B*b^3)*c
*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(
4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*
h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h
- 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^
2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/
3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^
2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h -
3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*
f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g +
 (d*e + c*f)*h)/(d*f*h))))/(d^4*f^4*h^4)
```

Sympy [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(a+bx)^2\,(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+e
)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt
(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x, algorithm="maxima")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*sq
rt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x, algorithm="giac")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*sq
rt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Hanged}$$

```
[In] int(((a + b*x)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2)*(g +
 h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.17 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 410

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$+\frac{2b^2\sqrt{-de+cf}(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(3abBdfh^2-3a^2Cdfh^2-b^2(3Bdfgh-C(ch(fg-eh)+dg(2fg+eh))))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+}{dg-}}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2/3*b^2*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*b^2*(3*B*d*f*
h-2*C*(c*f*h+d*e*h+d*f*g))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),
((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(
1/2)*(h*x+g)^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/
2)+2/3*(3*a*b*B*d*f*h^2-3*a^2*C*d*f*h^2-b^2*(3*B*d*f*g*h-C*(c*h*(-e*h+f*g)+
d*g*(e*h+2*f*g))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d
*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*
(h*x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.113, Rules used = {1629, 164, 115, 114, 122, 121}

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(-3a^2Cdfh^2+3abBdfh^2-(b^2(3Bdf\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$+\frac{2b^2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(3Bdfh-2C(cfh+deh+dfg))E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]),x]
```

```
[Out] (2*b^2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + (2*b^2*Sqrt
[-(d*e) + c*f]*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)*h^2*Sqrt[e
+ f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a*b*B*d*
f*h^2 - 3*a^2*C*d*f*h^2 - b^2*(3*B*d*f*g*h - c*C*h*(f*g - e*h) - C*d*g*(2*f
*g + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]
*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)
*h)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{2\int \frac{\frac{1}{2}d\left(3abBdfh-3a^2Cdfh-b^2C(deg+cfg+ceh)\right)+\frac{1}{2}b^2d(3Bdfh-2C(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3d^2fh}$$

$$
\begin{aligned}
&= \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} \\
&\quad + \frac{\left(b^2(3Bdfh-2C(dfg+deh+cfh))\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3dfh^2} + \frac{1}{3}\Bigg(3abB-3a^2C \\
&\quad -\frac{b^2(3Bdfgh-cCh(fg-eh)-Cdg(2fg+eh))}{dfh^2}\Bigg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} \\
&\quad + \frac{\left(\left(3abB-3a^2C-\frac{b^2(3Bdfgh-cCh(fg-eh)-Cdg(2fg+eh))}{dfh^2}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{3\sqrt{e+fx}} \\
&\quad + \frac{\left(b^2(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{3dfh^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} \\
&\quad + \frac{2b^2\sqrt{-de+cf}(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)\middle|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\
&\quad + \frac{\left(\left(3abB-3a^2C-\frac{b^2(3Bdfgh-cCh(fg-eh)-Cdg(2fg+eh))}{dfh^2}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{\cdots}{dg\cdots}}}}{3\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} \\
&\quad + \frac{2b^2\sqrt{-de+cf}(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)\middle|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\
&\quad + \frac{2\sqrt{-de+cf}(3abBdfh^2-3a^2Cdfh^2-b^2(3Bdfgh-cCh(fg-eh)-Cdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\cdots}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.00 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.08

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2b^2Cd^2fh(e+fx)(g+hx)+\frac{2b^2d^2(3Bdfh-2C(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx}+2ib^2\sqrt{-c+\frac{de}{f}}fh(3Bdfh\right.}{}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x
]*Sqrt[g + h*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(2*b^2*C*d^2*f*h*(e + f*x)*(g + h*x) + (2*b^2*d^2*(3*B*d*f*h
 - 2*C*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*x) + (2*I)*b^2*
Sqrt[-c + (d*e)/f]*f*h*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d
*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*Ell
ipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h
- c*f*h)] + ((2*I)*d*h*(3*a*b*B*d*f^2*h - 3*a^2*C*d*f^2*h + b^2*(-3*B*d*e*f
*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)))*Sqrt[c + d*x]*Sqrt[(d*(e
+ f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSin
h[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt
[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2Cb^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(abB-Ca^2-\frac{2Cb^2\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\right.}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x}}\right.}{}$
default	Expression too large to display

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/3*C*b^2/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2*(a*b*B-C*a^2-2/3*C*b^2/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*
d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+
e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f
*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(
-g/h+c/d))^(1/2))+2*(B*b^2-2/3*C*b^2/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*(
(x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1
/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/
d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^
(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.10

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}Cb^2d^2f^2h^2+(2Cb^2d^2f^2g^2+(Cb^2d^2ef+(Cb^2cd-3Bb^2d^2)f^2)gh+(2Cb^2d\right.}{}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*b^2*d^2*f^2*h^2 + (2*C*b
^2*d^2*f^2*g^2 + (C*b^2*d^2*e*f + (C*b^2*c*d - 3*B*b^2*d^2)*f^2)*g*h + (2*C
*b^2*d^2*e^2 + (C*b^2*c*d - 3*B*b^2*d^2)*e*f + (2*C*b^2*c^2 - 3*B*b^2*c*d -
 9*(C*a^2 - B*a*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*
f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2
*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*
e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c
^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e +
 c*f)*h)/(d*f*h)) + 3*(2*C*b^2*d^2*f^2*g*h + (2*C*b^2*d^2*e*f + (2*C*b^2*c*
d - 3*B*b^2*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (
d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2),
-4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c
*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2
+ 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^
2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4
/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d
^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 +
2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f
*h))))/(d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)\,(Bb - Ca + Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)
/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
+ h*x)), x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+
g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x + e
)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.18 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 291

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bC\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*b*C*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h
+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/
h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-B*b*h+C*a*h+C*b*g)
*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g)
)^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g)
)^(1/2)/d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{24, 164, 115, 114, 122, 121\}$

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(aCh-bBh+bCg)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ell
ipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]) - (2*Sqrt[-(d*e) + c*f]*(b*C*g - b*B*h + a*C*h)*Sqrt[(d*(e + f*x))/(d*e
- c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c +
d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*S
qrt[e + f*x]*Sqrt[g + h*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= \frac{(bC)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{h} - \frac{(bCg-bBh+aCh)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}$$

$$= -\frac{\left((bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{h\sqrt{e+fx}} + \frac{\left(bC\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{2bC\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{\left((bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2bC\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\left(bCd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)+ibC(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c}}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(b*C*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x) + I*b*C*(d*e - c*f)*h*(c
 + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c +
d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)] - I*d*(b*C*e - b*B*f + a*C*f)*h*(c + d*x)^(3/2)*Sqrt[(d
*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*Ar
cSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))
/(d^2*Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2(Bb-Ca)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2Cb\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+}{-\frac{g}{h}+}}}{\sqrt{dfh\,x^3+cf}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\left(BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bde\,h^2-BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bdfgh-CF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ade\,h^2+CF\left(\sqrt{\right.\right.}{}$

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*
x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2*(B*b-C*a)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*
((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x
+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/
f)/(-g/h+c/d))^(1/2))+2*C*b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-
g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d
*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h
)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g
/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.34

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(3\sqrt{dfh}Cbdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3d^2f^2h^2},-\frac{4\left(2d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\left(\right.\right.}{}\right.\right.}{}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(d*f*h)*C*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f +
 c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*
```

```
d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^
2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f
^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c
*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^
3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3
)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) + (
C*b*d*f*g + (C*b*d*e + (C*b*c + 3*(C*a - B*b)*d)*f)*h)*sqrt(d*f*h)*weierstr
assPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f
 + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2
*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3
- 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f
*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Bb - Ca + Cbx}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e
)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x
)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx + a)\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*sq
rt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx + a)\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*sq
rt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Hanged}$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.19 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 309

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2(bB-2aC)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*C*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d
*g))^(1/2)/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(B*b-2*C*a)*EllipticPi(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)
*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*
x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used

$= \{24, 1621, 175, 552, 551, 12, 122, 121\}$

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(bB-2aC)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e
 + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/
(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) -
 (2*(b*B - 2*a*C)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= \frac{\int \frac{b^2C}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2} + (bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= C\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx - (2(bB$$
$$-2aC))\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)$$

$$= \frac{\left(C\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{\sqrt{e+fx}}$$
$$- \frac{\left(2(bB-2aC)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}}$$

$$= \frac{\left(C\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{\left(2(bB-2aC)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{2(bB-2aC)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.06 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.81

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(-\left((bcC-bBd+aCd)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right), \frac{dfg-cfh}{deh-cfh}\right)\right)+(-bB+2aC)d}{(-bc+ad)\sqrt{-c+\frac{de}{f}}f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(-((b*c*C - b*B*d +
a*C*d)*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)]) + (-(b*B) + 2*a*C)*d*EllipticPi[-((b*c*f - a*d*f)/(b*d
*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/
(d*e*h - c*f*h)]))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f*x))/
(f*(c + d*x))]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2(Bb-2Ca)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x}{-\frac{g}{h}}}}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+c}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\left(B\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-gb)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)be\,h^2-B\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\right.\right.}{}$

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(
h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2*C*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)
/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x
+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h
+c/d))^(1/2))+2*(B*b-2*C*a)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/
(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2
+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g
/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**2/(d*x+c)**(1/2)/(f*
x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)*
sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)*
sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.20 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 680

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{b(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+\cdots}{(bc-ad)^2\sqrt{f}(be-af)(bg-\cdots}$$

```
[Out] -b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f
+b*e)/(-a*h+b*g)/(b*x+a)+b*(B*b-2*C*a)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f
-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*(
f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(f*
x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-(4*a^3*C*d*f*h+2*a*b^2*B*(c*f*h+d*e
*h+d*f*g)-b^3*(B*d*e*g-c*(-B*e*h-B*f*g+2*C*e*g))-a^2*b*(3*B*d*f*h+2*C*(c*f*
h+d*e*h+d*f*g)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+
d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*
x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)
/(-a*h+b*g)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-(B*b-2*C*a)*EllipticF(f^(1/
2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)
*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/
(-a*d+b*c)/(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.183$, Rules used = {24, 1613, 1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$
$$-\frac{\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2}$$
$$-\frac{\sqrt{f}(bB-2aC)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{e+fx}\sqrt{g+hx}(bc-ad)(be-af)}$$
$$+\frac{b\sqrt{f}\sqrt{g+hx}(bB-2aC)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{e+fx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e
 + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] -((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) + (b*(b*B - 2*a*C)*Sqrt[f]*Sqrt[-(d*e
) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sq
rt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])
/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g
 - c*h)]) - ((b*B - 2*a*C)*Sqrt[f]*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d
*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c
 + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d
)*(b*e - a*f)*Sqrt[e + f*x]*Sqrt[g + h*x]) - (Sqrt[-(d*e) + c*f]*(4*a^3*C*d
*f*h + 2*a*b^2*B*(d*f*g + d*e*h + c*f*h) - b^3*(B*d*e*g - c*(2*C*e*g - B*f*
g - B*e*h)) - a^2*b*(3*B*d*f*h + 2*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[(d*(e +
 f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e -
c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
 ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*(b*e - a*f)*(b*g
- a*h)*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\int \frac{b^2\left(b^2C(2bceg-a(deg+cfg+ceh))-(bB-aC)\left(2a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh)\right)\right)+2ab^3(bB-2aC)dfhx+b^4(bB-}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2b^2(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\int \frac{ab^3Bdfh-2a^2b^2Cdfh+(b^4Bdfh-2ab^3Cdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b^2(bc-ad)(be-af)(bg-ah)} + \frac{(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+2C}{2(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} - \frac{((bB-2aC)df)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2(bc-ad)(be-af)} + \frac{(b(bB-2aC)df)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2(bc-ad)(be-af)(bg-ah)} - \frac{(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+2C}{(bc-ad)(be-af)(bg}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} - \frac{\left((bB-2aC)df\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{2(bc-ad)(be-af)\sqrt{e+fx}} - \frac{\left((4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+2\right.}{(bc-ad)(be-af)(} + \frac{\left(b(bB-2aC)df\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{2(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{b(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{\left((bB-2aC)df\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{2(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$

$$- \frac{\left((4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+2C\right.}{(bc-ad)(be-af)(b}$$

$$= -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{b(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$

$$- \frac{\sqrt{-de+cf}(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3I}{(bc-ad)^2\sqrt{f}(be-af)(bg-}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.66 (sec) , antiderivative size = 3419, normalized size of antiderivative = 5.03

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] -((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(b^3*B*c*Sqrt[-c +
 (d*e)/f]*f*h - 2*a*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*b^2*B*d*Sqrt[-c + (d
*e)/f]*f*h + 2*a^2*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (b^3*B*c*d^2*e*Sqrt[-c +
(d*e)/f]*g)/(c + d*x)^2 - (2*a*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x
)^2 - (a*b^2*B*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (2*a^2*b*C*d^3*e*S
qrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (b^3*B*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c
```

```
+ d*x)^2 + (2*a*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*b^2*B*
c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*Sqrt[-c + (d*e
)/f]*f*g)/(c + d*x)^2 - (b^3*B*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 +
(2*a*b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2*B*c*d^2*e*Sqr
t[-c + (d*e)/f]*h)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h)/(
c + d*x)^2 + (b^3*B*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (2*a*b^2*c^3*
C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^2*d*Sqrt[-c + (d*e)/f]*f
*h)/(c + d*x)^2 + (2*a^2*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (b
^3*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (2*a*b^2*c*C*d*Sqrt[-c + (d*e)
/f]*f*g)/(c + d*x) - (a*b^2*B*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (2*a^
2*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (b^3*B*c*d*e*Sqrt[-c + (d*e)/
f]*h)/(c + d*x) - (2*a*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*b^2
*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) + (2*a^2*b*C*d^2*e*Sqrt[-c + (d*e)
/f]*h)/(c + d*x) - (2*b^3*B*c^2*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (4*a*b^
2*c^2*C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (2*a*b^2*B*c*d*Sqrt[-c + (d*e)/
f]*f*h)/(c + d*x) - (4*a^2*b*c*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (I*b
*(b*B - 2*a*C)*(-(b*c) + a*d)*(-(d*e) + c*f)*h*Sqrt[1 - c/(c + d*x) + (d*e)
/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticE[I*Arc
Sinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/S
qrt[c + d*x] + (I*d*(2*a*b*B*d*f - 2*a^2*C*d*f + b^2*(2*c*C*e - B*d*e - B*c
*f))*(-(b*g) + a*h)*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/
(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqr
t[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*b^3*c
*C*d*e*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) +
 (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcS
inh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sq
rt[c + d*x] - (I*b^3*B*d^2*e*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*
Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b
*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h
)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*b^3*B*c*d*f*g*Sqrt[1 - c/(c + d*x) +
 (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticP
i[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
 d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*a*b^2*B*d^
2*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d
*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh
[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[
c + d*x] - ((2*I)*a^2*b*C*d^2*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x)
)]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)
/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*
f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*b^3*B*c*d*e*h*Sqrt[1 - c/(c + d*x
) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*Ellipt
icPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[
c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*a*b^2*B
*d^2*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) +
 (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcS
```

```
inh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sq
rt[c + d*x] - ((2*I)*a^2*b*C*d^2*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d
*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d
*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g -
 c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*a*b^2*B*c*d*f*h*Sqrt[1 - c
/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x)
)]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)
/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((2*I
)*a^2*b*c*C*d*f*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c
 + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)
), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*
f*h)])/Sqrt[c + d*x] - ((3*I)*a^2*b*B*d^2*f*h*Sqrt[1 - c/(c + d*x) + (d*e)/
(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*
c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
 (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((4*I)*a^3*C*d^2*f*h*Sqr
t[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c
 + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c
+ (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x])
)/(d*(b*c - a*d)*(-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*(-(b*e) + a*f)*(-(b*g) +
 a*h)*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g + ((c + d*x)*(h
- (c*h)/(c + d*x)))/d])
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 1211, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1211
default	Expression too large to display	13369

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(
h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
b^2/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2
+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(B*b-2*C*a)/(a^3*d*f*
h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b
^3*c*e*g)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((
x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c
*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)
/(-g/h+c/d))^(1/2))-d*f*h*b*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(g/h-e/f)*((x+g/
h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(
d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2
```

```
)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(
1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)
))+(3*B*a^2*b*d*f*h-2*B*a*b^2*c*f*h-2*B*a*b^2*d*e*h-2*B*a*b^2*d*f*g+B*b^3*c
*e*h+B*b^3*c*f*g+B*b^3*d*e*g-4*C*a^3*d*f*h+2*C*a^2*b*c*f*h+2*C*a^2*b*d*e*h+
2*C*a^2*b*d*f*g-2*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f
*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(g/h-e/f)*((x+g/h)/(g/h
-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x
^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h
+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f
)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**3/(d*x+c)**(1/2)/(f*
x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)*
sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)*
sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.21 $\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 980

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b(4bBdfh+C(adfh-3b(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} + \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} - \frac{b\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+C(adfh-3b(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\right.\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{(be-af)\sqrt{bg-ah}(aCdfh-b(4Bdfh-C(3dfg+3deh+cfh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\right.}{4df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} - \frac{\sqrt{-dg+ch}((adfh+b(dfg+deh+cfh))(4bBdfh+C(adfh-3b(dfg+deh+cfh)))+4dfh(2a^2Cdf}{}$$

```
[Out] -1/4*((a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(4*b*B*d*f*h+C*(a*d*f*h-3*b*(c*f*h+d*
e*h+d*f*g)))+4*d*f*h*(2*a^2*C*d*f*h+b^2*C*(c*e*h+c*f*g+d*e*g)-a*b*(4*B*d*f*
h-C*(c*f*h+d*e*h+d*f*g))))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2
)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*
h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-
c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/d^2/f
^2/h^3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*b*(4*b*B*d*f*h+C*(a
*d*f*h-3*b*(c*f*h+d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/
d/f^2/h^2/(d*x+c)^(1/2)+1/2*b^2*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)
```

```
*(h*x+g)^(1/2)/d/f/h+1/4*(-a*f+b*e)*(a*C*d*f*h-b*(4*B*d*f*h-C*(c*f*h+3*d*e*
h+3*d*f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x
+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^
(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/d/f^2/h^2
/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1
/2)-1/4*b*(4*b*B*d*f*h+C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g)))*EllipticE((-c*h
+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+
f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a
)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d^2/f^2/h^2/((-c*f+d
*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.89 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.145$, Rules used = {1614, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg}}{\sqrt{fg}}\right.\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+\frac{(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}b}{4df^2h^2\sqrt{c+dx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(4bBdfh-aCdfh-bC(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\text{arcsi}\right.}{4df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$-\frac{\sqrt{ch-dg}((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+4dfh(2Cdfha^2-}{}$$

```
[In] Int[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (b*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*
Sqrt[e + f*x]*Sqrt[g + h*x])/(4*d*f^2*h^2*Sqrt[c + d*x]) + (b^2*C*Sqrt[a +
b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) - (b*Sqrt[d*g - c
*h]*Sqrt[f*g - e*h]*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h
))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*E
llipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d
*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(4*d^2*f^2*h^2
*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((b
*e - a*f)*Sqrt[b*g - a*h]*(4*b*B*d*f*h - a*C*d*f*h - b*C*(c*f*h + 3*d*(f*g
+ e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
```

```
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
+ b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(4*d*f^2
*h^2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e
*h)*(a + b*x)))]) - (Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*
h))*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h)) + 4*d*f*h*(2*
a^2*C*d*f*h + b^2*C*(d*e*g + c*f*g + c*e*h) - a*b*(4*B*d*f*h - C*(d*f*g + d
*e*h + c*f*h))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b
*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b
*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqr
t[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*
g - e*h))])/(4*d^2*Sqrt[b*c - a*d]*f^2*h^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

<u>Rule 171</u>

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

<u>Rule 176</u>

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

<u>Rule 182</u>

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

<u>Rule 430</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
```

```
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))-2b\left(2a^2Cdfh+b^2C(deg+cfg+ceh)-ab(4Bdfh-C(dfg+deh+cfh))\right)x+b^2(4bBdfh+aCdf \cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh}$$

$$= \frac{b(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} + \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{-b\left(b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)\right)-b^2((adfh+ \cdots}{\sqrt{a+bx}\sqrt{c+ \cdots}}}{8b \cdots} + \frac{(b(de-cf)(dg-ch)(4bBdfh+aCdfh-3bC(dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8d^2f^2h^2}$$

$$= \frac{b(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}} + \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} - \frac{((be-af)(bg-ah)(4bBdfh-aCdfh-bC(cfh+3d(fg+eh))))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8df^2h^2} - \frac{((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+4dfh(2a^2Cdfh+ \cdots}{8d^2f^2h^2} - \frac{\left(b(dg-ch)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{\cdots}}{\sqrt{\cdots}} \cdots\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{b(4bBdfh + aCdfh - 3bC(dfg + deh + cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$
$$+ \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{b\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh + aCdfh - 3bC(dfg + deh + cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\Big(\sin}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$- \frac{\Big(((adfh + b(dfg + deh + cfh))(4bBdfh + aCdfh - 3bC(dfg + deh + cfh)) + 4dfh(2a^2Cdfh +}{}$$
$$- \frac{\Big((be-af)(bg-ah)(4bBdfh - aCdfh - bC(cfh + 3d(fg + eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\Big)\text{ Subs}}{4df^2h^2(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$= \frac{b(4bBdfh + aCdfh - 3bC(dfg + deh + cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$
$$+ \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{b\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh + aCdfh - 3bC(dfg + deh + cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\Big(\sin}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$- \frac{(be-af)\sqrt{bg-ah}(4bBdfh - aCdfh - bC(cfh + 3d(fg + eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\Big(\sin^{-1}}{4df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$- \frac{\sqrt{-dg+ch}((adfh + b(dfg + deh + cfh))(4bBdfh + aCdfh - 3bC(dfg + deh + cfh)) + 4dfh(2}{}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. 2(980) = 1960.

Time = 36.91 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.41

$$\int \frac{\sqrt{a+bx}(abB - a^2C + b^2Bx + b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. 2(897) = 1794.

Time = 5.28 (sec) , antiderivative size = 1834, normalized size of antiderivative = 1.87

method	result	size
elliptic	Expression too large to display	1834
default	Expression too large to display	56432

```
[In] int((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(1/2*C*b^2/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+
b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c
*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)
+2*(a^2*b*B-C*a^3-1/2*C*b^2/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*
b*c*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2
*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/
b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x
+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-
c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b^2*B-C*a^2*b-1/2*C*b^2
/d/f/h*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+
a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d
)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*Elliptic
F(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+
e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)
/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d
+g/h))^(1/2)))+(B*b^3+C*b^2*a-1/2*C*b^2/d/f/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*
b*d*e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b
)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d
))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/
h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/
b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+
e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/
h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*
d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d)
)^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1
/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)
/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((a + b*x)**(3/2)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2
)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2
)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.22 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 734

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{bC\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{C(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} - \frac{\sqrt{-dg+ch}(aCdfh-b(2Bdfh-C(dfg+deh+cfh)))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}}{d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -(a*C*d*f*h-b*(2*B*d*f*h-C*(c*f*h+d*e*h+d*f*g)))*(b*x+a)*EllipticPi((-a*d+b
*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b
*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*
((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g
)/(b*x+a))^(1/2)/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+b*C*(
b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h/(d*x+c)^(1/2)-C*(-a*f+b*e)*Ell
ipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*
d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((-a*f+b*e
)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/f/h/(-e*h+f*g)^(1/2)/(d*x
+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-b*C*EllipticE((-c*
h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h
+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+
a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d/f/h/((-c*f+d*e)*(
b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1600, 1610, 176, 430, 182, 435, 171, 551}

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\,\text{EllipticPi}\left(-\right.}{dfh^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$-\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$-\frac{bC\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}$$

$$+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (b*C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (b*C*
Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x)
)/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x]
)/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*
(d*g - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]
*Sqrt[g + h*x]) - (C*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]
*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h
))/((d*e - c*f)*(b*g - a*h)))])/(f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-((
(b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[-(d*g) + c*h]*(2*
b*B*d*f*h - a*C*d*f*h - b*C*(d*f*g + d*e*h + c*f*h))*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[
(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e
- a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*Sqrt[b*c - a*d]*f*h^2*Sq
rt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
```

```
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1610

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Dis
t[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[B*(d*e - c*f)*((d*g - c*h)/(2*
d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x
], x] + Dist[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b*d
*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e + c*
f), 0]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+bx}(bB-aC+bCx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{(C(be-af)(bg-ah))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2fh}$$
$$+ \frac{(bC(de-cf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2dfh}$$
$$+ \frac{(2b(bB-aC)dfh+bC(adfh-b(dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bdfh}$$

$$= \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$+ \frac{\left((2b(bB-aC)dfh+bC(adfh-b(dfg+deh+cfh)))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Su}}{bdfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$- \frac{\left(C(be-af)(bg-ah)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{fh(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$- \frac{\left(bC(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$- \frac{bC\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$- \frac{C(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+ \frac{\sqrt{-dg+ch}(2bBdfh-aCdfh-bC(dfg+deh+cfh))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\Big(-}{d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8107 vs. 2(734) = 1468.

Time = 42.83 (sec) , antiderivative size = 8107, normalized size of antiderivative = 11.04

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(669) = 1338.

Time = 4.90 (sec) , antiderivative size = 1552, normalized size of antiderivative = 2.11

method	result	size
elliptic	Expression too large to display	1552
default	Expression too large to display	20101

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(B*a*b-C*a^2)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a
/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*
((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*
h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*
B*b^2*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-
c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(
x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h
))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/
f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/
h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+C*b^2*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a
/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+
e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2
)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h
+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c
/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d)
)^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*
f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a
/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a
/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+bx}(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(1/2)/(d*x+c)**(1/2)
/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

Maxima [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.23 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 436

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bB-2aC)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{}}{\sqrt{fg-eh}\sqrt{}}\right.\right.}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+}{(fg-eh)(a+}}}$$
$$+\frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)(dg}{(bc-ad)(fg}\right.}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 2*C*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+
a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h
+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)
*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/h/(-a*d+b*c)^(1/2)/(d*x+c)^(
1/2)/(f*x+e)^(1/2)+2*(B*b-2*C*a)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(
-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g)
)^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*h+
b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(
b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{24, 1612, 176, 430, 171, 551\}$

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}}{\sqrt{fg-eh}}\right.\right.}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be}{(a+bx)(fg}}}$$

$$+\frac{2C(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\mathrm{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-c}{(bc-ad)(fg-e}\right.}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(b*B - 2*a*C)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt
[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/
(Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x
))/((f*g - e*h)*(a + b*x)))]) + (2*C*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*
g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/(
(f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcS
in[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b
*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
```

```
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= C\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx + (bB-2aC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\left(2C(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g+hx}}{\sqrt{a+bx}}\right)}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$+ \frac{\left(2(bB-2aC)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2(bB-2aC)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+\frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-af)(\cdots}{(bc-ad)(\cdots}\right.}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [A] (verified)

Time = 25.18 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.34

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{bB\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\text{arcs}\cdots\right.}{(bg-ah)(a+bx)\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-
((b*B*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ellip
ticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(
b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a + b
*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (2*a*C*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*EllipticF[ArcS
in[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*
d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a + b*x)*Sq
rt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (C*(-(f*g) + e*h)
*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b
*x)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[Sqrt[((-(b*e
) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*
h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(Sqrt[c + d*x]*Sqrt[e + f
*x]*Sqrt[g + h*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(398) = 796.

Time = 6.31 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.96

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2(Bb-Ca)\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)(x+}{\left(-\frac{g}{h}+\frac{a}{b}\right)(x+}}\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.}{}$
default	Expression too large to display

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(B*b-C*a)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c
/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x
+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/
b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*C*b*
(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/
b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d)
)^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)
*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bb - Ca + Cbx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(3/2)/(d*x+c)**(1/2)
/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((B*b - C*a + C*b*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.24 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 616

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2b(bB-2aC)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{2(bcC-bBd+aCd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[Out] 2*b*(B*b-2*C*a)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*
f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(1/
2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*(-B*b*d+C
*a*d+C*b*c)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+
a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*
(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)
/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1
/2)-2*b*(B*b-2*C*a)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/
2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h
+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/
(d*x+c))^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d*e)*(b*x+a)/(-a*f+b
*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {24, 1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{g+hx}(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\right.}$$
$$-\frac{2b\sqrt{a+bx}(bB-2aC)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{\sqrt{g+hx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}$$
$$-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}+\frac{2bd\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{c+dx}(bc-ad)(be-af)(bg-ah)}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*(b*B - 2*a*C)*d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*Sqrt[c + d*x]) - (2*b^2*(b*B - 2*a*C)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a
 + b*x]) - (2*b*(b*B - 2*a*C)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]
*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[
(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a
*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/((b*c - a*d)*(b*e - a*f)*(b*g
- a*h)*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x])
 + (2*(b*c*C - b*B*d + a*C*d)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sq
rt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*
g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqr
t[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= -\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{\int \frac{b^2\left(b^2C(bceg-a(deg+cfg+ceh))-a(bB-aC)(adfh-b(dfg+deh+cfh))\right)+b^3(bB-2aC)(adfh+b(dfg+deh+cfh))x+2b^4(bB-2aC)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{b^2(bc-ad)(be-af)(bg-ah)}$$

$$= \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{\int \frac{2b^3d(bcC-bBd+aCd)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b^3d(bc-ad)f(be-af)h(bg-ah)} + \frac{(b(bB-2aC)(de-cf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(be-af)(bg-ah)}$$

$$= \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{(bcC-bBd+aCd)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{\left(2b(bB-2aC)(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2b(bB-2aC)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-e}{(be-af)(dg-c}\right.}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{\left(2(bcC-bBd+aCd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2b(bB-2aC)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-e}{(be-af)(dg-c}\right.}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{2(bcC-bBd+aCd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] (verified)

Time = 26.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.55

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left(b(bB-2aC\right.}{}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e + f
*x)^(3/2)*(g + h*x)^(3/2)*(b*(b*B - 2*a*C)*(d*g - c*h)*EllipticE[ArcSin[Sqr
t[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g -
 e*h))/((b*e - a*f)*(d*g - c*h))] + (b*c*C - b*B*d + a*C*d)*(b*g - a*h)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((
b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*g - e*
h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g
 + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2248 vs. 2(562) = 1124.

Time = 7.51 (sec) , antiderivative size = 2249, normalized size of antiderivative = 3.65

method	result	size
elliptic	Expression too large to display	2249
default	Expression too large to display	18867

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*
c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)/((x+a
/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*
d*e*g*x+b*c*e*g))^(1/2)+2*(C+(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c
*e*h+b^2*c*f*g+b^2*d*e*g)*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^
2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b*c*f*g+b
*d*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*
c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g
/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1
/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*
d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)
)+2*(-b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*
h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2
*b*c*f*h+2*b*d*e*h+2*b*d*f*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a))*(g/h-a/b)*(
(-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/
(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g
/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*El
lipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/
(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)
/(-c/d+g/h))^(1/2)))-2*d*f*h*b^2*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d
*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*((x+a/b)*(x
+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/
d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/
h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a
/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h
-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b
)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/
2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*Ellipti
```

```
cPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/
f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(
x+e/f)*(x+g/h))^(1/2))
```

Fricas [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt
(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a
*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e
 + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*
f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*
a*b*c + a^2*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(5/2)/(d*x+c)**(1/2)
/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

3.25 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 1128

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bd(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2(\cdots}{\cdots}$$

$$-\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2b^2(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh\cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+\cdots}}$$

$$-\frac{2b\sqrt{dg-ch}\sqrt{fg-eh}(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2(C(deg+cfg+ceh)+4\cdots}{3(bc-ad)^2(be-\cdots}$$

$$-\frac{2(3a^3Cd^2fh-b^3(2Bd^2eg-Bc^2fh-cd(3Ceg-Bfg-Beh))-3a^2bd(Bdfh+C(dfg+deh-cfh))+\cdots}{3(bc-ad)^2(be-af)(\cdots}$$

```
[Out] 2/3*b*d*(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))+a*b^2*(
C*(c*e*h+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*h+
d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f
+b*e)^2/(-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e
)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*b^
2*(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))+a*b^2*(C*(c*e
*h+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*h+d*e*h+
d*f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^
2/(-a*h+b*g)^2/(b*x+a)^(1/2)-2/3*(3*a^3*C*d^2*f*h-b^3*(2*B*d^2*e*g-B*c^2*f*
h-c*d*(-B*e*h-B*f*g+3*C*e*g))-3*a^2*b*d*(B*d*f*h+C*(-c*f*h+d*e*h+d*f*g))+a*
b^2*(3*B*d^2*(e*h+f*g)+C*(-2*c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)))*EllipticF((
-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(
```

```
-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x
+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(3/2)/(-e*h+f*g
)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2/3*b*
(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))+a*b^2*(C*(c*e*h
+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*h+d*e*h+d*
f*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1
/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(
-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2
)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*
x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.99 (sec) , antiderivative size = 1119, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {24, 1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2(9Cdfha^3 - b(6Bdfh + 5C(dfg + deh + cfh))a^2 + b^2(C(deg + cfg + ceh) + 4B(dfg + deh + cfh))a + b}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$-\frac{2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}(9Cdfha^3 - b(6Bdfh + 5C(dfg + deh + cfh))a^2 + b^2(C(deg + cfg + ceh) + 4B(dfg}{3(bc-ad)^2(be-a}$$

$$+\frac{2d(9Cdfha^3 - b(6Bdfh + 5C(dfg + deh + cfh))a^2 + b^2(C(deg + cfg + ceh) + 4B(dfg + deh + cfh))a +}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

$$-\frac{2(3Cd^2fha^3 - 3bd(Bdfh + C(dfg + deh - cfh))a^2 + b^2(3B(fg+eh)d^2 + C(-2fhc^2 - dfgc - dehc + d^2}{3(bc-ad)^2(be-af)}$$

```
[In] Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*d*(9*a^3*C*d*f*h + b^3*(3*c*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a
*b^2*(C*(d*e*g + c*f*g + c*e*h) + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B
*d*f*h + 5*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[c + d*x]) - (2*b^2
*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(b
*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - (2*b^2*(9*a^3*C*d*f*h + b^3*(3*c*C
*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g + c*e*h) +
4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g + d*e*h + c*f*
```

```
h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)^2*(b*e - a*f
)^2*(b*g - a*h)^2*Sqrt[a + b*x]) - (2*b*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(9*
a^3*C*d*f*h + b^3*(3*c*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d
*e*g + c*f*g + c*e*h) + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5
*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/(
(f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(
Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*
g - c*h))])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[((d*e - c*f)*
(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*(3*a^3*C*d^2*f*h -
b^3*(2*B*d^2*e*g - B*c^2*f*h - c*d*(3*C*e*g - B*f*g - B*e*h)) - 3*a^2*b*d*(
B*d*f*h + C*(d*f*g + d*e*h - c*f*h)) + a*b^2*(3*B*d^2*(f*g + e*h) + C*(d^2*
e*g - c*d*f*g - c*d*e*h - 2*c^2*f*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e -
 c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e +
f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e -
 c*f)*(b*g - a*h)))])/(3*(b*c - a*d)^2*(b*e - a*f)*(b*g - a*h)^(3/2)*Sqrt[f
*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*
x)))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
```

```
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b^2(bB-aC)+b^3Cx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= -\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$+ \frac{\int \frac{b^2(b^2C(3bceg-a(deg+cfg+ceh))-(bB-aC)(3a^2dfh+2b^2(deg+cfg+ceh)-3ab(dfg+deh+cfh)))+b^3(bB-2aC)(3adfh-b(dfg-}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3b^2(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$- \frac{2b^2(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+de}{3(bc-ad)^2(be-af)^2(bg-a}$$

$$+ \frac{\int \frac{b^2(b^2(bB-2aC)(bceg-a(deg+cfg+ceh))(3adfh-b(dfg+deh+cfh))-a(adfh-b(dfg+deh+cfh))(b^2C(3bceg-a(deg+cfg+ceh)}{}}{}$$

$$= \frac{2bd(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+deh}{3(bc-ad)^2(be-af)^2(bg-ah)}$$

$$- \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$- \frac{2b^2(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+de}{3(bc-ad)^2(be-af)^2(bg-a}$$

$$+ \frac{\int -\frac{2b^3df(be-af)h(bg-ah)(3a^3Cd^2fh-3a^2bd(Bdfh+C(dfg+deh-cfh))-b^3(2Bd^2eg-Bc^2fh-cd(3Ceg-B(fg+eh)))+ab^2(3B}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6b^3d(bc-ad)^2f(be-af)^2h(bg-ah)^2}$$

$$+ \frac{(b(de-cf)(dg-ch)(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+c}{3(bc-ad)^2(be-}$$

$$= \frac{2bd(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+deh}{3(bc-ad)^2(be-af)^2(bg-ah)}$$

$$- \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$- \frac{2b^2(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+de}{3(bc-ad)^2(be-af)^2(bg-a}$$

$$- \frac{(3a^3Cd^2fh-3a^2bd(Bdfh+C(dfg+deh-cfh))-b^3(2Bd^2eg-Bc^2fh-cd(3Ceg-B(fg+}{3(bc-ad)^2(be-}$$

$$- \frac{(2b(dg-ch)(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4}{3(bc-ad)^2(}$$

$$= \frac{2bd(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ceh) + 4B(dfg+deh+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\cdot}$$
$$- \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b^2(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ceh) + 4B(dfg+deh}{3(bc-ad)^2(be-af)^2(bg-ah}$$
$$- \frac{2b\sqrt{dg-ch}\sqrt{fg-eh}(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ce}{3(bc-ad}$$
$$- \frac{\left(2(3a^3Cd^2fh - 3a^2bd(Bdfh + C(dfg+deh-cfh)) - b^3(2Bd^2eg - Bc^2fh - cd(3Ceg - B(fg\right.}{3(bc-ad)}$$

$$= \frac{2bd(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ceh) + 4B(dfg+deh+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\cdot}$$
$$- \frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$- \frac{2b^2(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ceh) + 4B(dfg+deh}{3(bc-ad)^2(be-af)^2(bg-ah}$$
$$- \frac{2b\sqrt{dg-ch}\sqrt{fg-eh}(9a^3Cdfh + b^3(3cCeg - 2Bdeg - 2Bc(fg+eh)) + ab^2(C(deg+cfg+ce}{3(bc-ad}$$
$$- \frac{2(3a^3Cd^2fh - b^3(2Bd^2eg - Bc^2fh - cd(3Ceg - Bfg - Beh)) - 3a^2bd(Bdfh + C(dfg+deh-}{3(bc-ad)^2(be-}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10836 vs. 2(1128) = 2256.

Time = 40.01 (sec) , antiderivative size = 10836, normalized size of antiderivative = 9.61

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3424 vs. 2(1056) = 2112.

Time = 10.27 (sec) , antiderivative size = 3425, normalized size of antiderivative = 3.04

method	result	size
elliptic	Expression too large to display	3425
default	Expression too large to display	110289

```
[In] int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/
2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*
b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(b*d*f*h*x^4+a*d*f
*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*
x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e
*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*
f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2
*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(6*B*
a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g+2*B*b^3*c*e*h+2
*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*C*a^2*b*d*e*h+5*
C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C*b^3*c*e*g)/((x+
a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b
*d*e*g*x+b*c*e*g))^(1/2)+2*(-1/3*(3*B*a*b*d*f*h-B*b^2*c*f*h-B*b^2*d*e*h-B*b
^2*d*f*g-6*C*a^2*d*f*h+2*C*a*b*c*f*h+2*C*a*b*d*e*h+2*C*a*b*d*f*g)/(a^3*d*f*
h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b
^3*c*e*g)+1/3*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+
b^2*d*e*g)*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g
+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*
C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C
*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^
2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2-1/3*(b*c*e*h+b*c*f*g+b*d*e*g)*b/(a^3*d*f*h
-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^
3*c*e*g)^2*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g
+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*
C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C
*b^3*c*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d
)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h
+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)
*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e
/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(-1/3*b*(a*d*f*h-b*c*f*h-
b*d*e*h-b*d*f*g)*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2
*d*f*g+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*
f*h+5*C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e
```

```
*g-3*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*
h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2-1/3*(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)*
b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*
b^2*d*e*g-b^3*c*e*g)^2*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B
*a*b^2*d*f*g+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^
2*b*c*f*h+5*C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b
^2*d*e*g-3*C*b^3*c*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^
(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(
x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+
c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)
*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+
c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))-2/3*d*f*h*b^2*(6*B
*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g+2*B*b^3*c*e*h+
2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*C*a^2*b*d*e*h+5
*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C*b^3*c*e*g)/(a^
3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d
*e*g-b^3*c*e*g)^2*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-
g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(
1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/
d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(
x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)
*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*
g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1
/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))
))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt
(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a
*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^
2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b
^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)
*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3
*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e
)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(7/2)/(d*x+c)**(1/2)
/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x +
 c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*
(a + b*x)^(7/2)*(c + d*x)^(1/2)), x)
```

3.26 $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 1097

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(4C(2adfh-3b(dfg+deh+cfh))(adfh-2b(dfg+deh+cfh))+5bdfh(7Abdfh-C(5b(deg+cfg+c}{105d^3f^3h^3}$$

$$+\frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2}$$

$$+\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}$$

$$-\frac{4\sqrt{-de+cf}(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+d^2}{}$$

$$+\frac{2\sqrt{-de+cf}(35a^2d^2f^2h^2(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))-14abdfh(15Ad^2f^2gh^2+C(4c^2f}{}$$

```
[Out] 2/105*(4*C*(2*a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h-2*b*(c*f*h+d*e*h+d*
f*g))+5*b*d*f*h*(7*A*b*d*f*h-C*(5*b*(c*e*h+c*f*g+d*e*g)+2*a*(c*f*h+d*e*h+d*
f*g))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^3/f^3/h^3+4/35*C*(2*a*d
*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(
1/2)/d^2/f^2/h^2+2/7*C*(b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/
d/f/h-4/105*(35*a^2*C*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)-7*a*b*d*f*h*(15*A*d^2
*f^2*h^2+C*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^
2*g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)+2*C*(12*c^3*f^3*h^3+10*c
^2*d*f^2*h^2*(e*h+f*g)+c*d^2*f*h*(10*e^2*h^2+9*e*f*g*h+10*f^2*g^2)+2*d^3*(6
*e^3*h^3+5*e^2*f*g*h^2+5*e*f^2*g^2*h+6*f^3*g^3))))*EllipticE(f^(1/2)*(d*x+c
```

```
)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*
(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^4/f^(7/2)/h^4/(f*x+e)^(1/2)/(d
*(h*x+g)/(-c*h+d*g))^(1/2)+2/105*(35*a^2*d^2*f^2*h^2*(3*A*d*f*h^2+C*(c*h*(-
e*h+f*g)+d*g*(e*h+2*f*g)))-14*a*b*d*f*h*(15*A*d^2*f^2*g*h^2+C*(4*c^2*f*h^2*
(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*g*h+
8*f^2*g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))+C*(24*c
^3*f^2*h^3*(-e*h+f*g)+c^2*d*f*h^2*(-23*e^2*h^2+6*e*f*g*h+17*f^2*g^2)+2*c*d^
2*h*(-12*e^3*h^3+3*e^2*f*g*h^2+e*f^2*g^2*h+8*f^3*g^3)+d^3*g*(24*e^3*h^3+17*
e^2*f*g*h^2+16*e*f^2*g^2*h+48*f^3*g^3))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(
c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e
)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^4/f^(7/2)/h^4/(f*x+e)^(1
/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 1083, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1615, 1614, 1629, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a+bx)^2}{7dfh}$$
$$+\frac{4C(2adfh-3b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a+bx)}{35d^2f^2h^2}$$
$$-\frac{4\sqrt{cf-de}((35Ad^2f^2(dfg+deh+cfh)h^2+2C(2(6f^3g^3+5ef^2hg^2+5e^2fh^2g+6e^3h^3)d^3+cfh(10f\cdots}{\cdots}$$
$$+\frac{2\sqrt{cf-de}((35Ad^2f^2(ch(fg-eh)+dg(2fg+eh))h^2+C(g(48f^3g^3+16ef^2hg^2+17e^2fh^2g+24e^3h^3\cdots}{\cdots}$$
$$+\frac{2\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)\cdots}{105d^2f^2h^2}$$

```
[In] Int[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] (2*(35*A*b^2*d*f*h + 8*a^2*C*d*f*h - 25*b^2*C*(d*e*g + c*f*g + c*e*h) - 38*
a*b*C*(d*f*g + d*e*h + c*f*h) + (24*b^2*C*(d*f*g + d*e*h + c*f*h)^2)/(d*f*h
))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(105*d^2*f^2*h^2) + (4*C*(2*a
*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]
*Sqrt[g + h*x])/(35*d^2*f^2*h^2) + (2*C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e +
f*x]*Sqrt[g + h*x])/(7*d*f*h) - (4*Sqrt[-(d*e) + c*f]*(35*a^2*C*d^2*f^2*h^2
*(d*f*g + d*e*h + c*f*h) - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2
 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*
```

```
(35*A*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*
f^2*h^2*(f*g + e*h) + c*d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d
^3*(6*f^3*g^3 + 5*e*f^2*g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*Sqrt[(d*(e +
f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(105*d^4*f^(7/2)*h^4*
Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(35*
a^2*d^2*f^2*h^2*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - 1
4*a*b*d*f*h*(15*A*d^2*f^2*g*h^2 + C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2
*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))) +
 b^2*(35*A*d^2*f^2*h^2*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) + C*(24*c^3*f^
2*h^3*(f*g - e*h) + c^2*d*f*h^2*(17*f^2*g^2 + 6*e*f*g*h - 23*e^2*h^2) + 2*c
*d^2*h*(8*f^3*g^3 + e*f^2*g^2*h + 3*e^2*f*g*h^2 - 12*e^3*h^3) + d^3*g*(48*f
^3*g^3 + 16*e*f^2*g^2*h + 17*e^2*f*g*h^2 + 24*e^3*h^3))))*Sqrt[(d*(e + f*x)
)/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(105*d^
4*f^(7/2)*h^4*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
```

```
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f*
g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
```

```
2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} + \frac{\int \frac{(a+bx)(-4bcCeg+7aAdfh-aC(deg+cfg+ceh)+(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2C(2adfh-3b(dfg+deh+cfh))x^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{7dfh}$$

$$= \frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2} + \frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} + \frac{\int \frac{-5adfh(4bcCeg-7aAdfh+aC(deg+cfg+ceh))-2C(2bceg+a(deg+cfg+ceh))(2adfh-3b(dfg+deh+cfh))-2\big(C(3b(deg+cfg+ce}{}}{}$$

$$= \frac{2\Big(35Ab^2dfh+8a^2Cdfh-25b^2C(deg+cfg+ceh)-38abC(dfg+deh+cfh)+\frac{24b^2C(dfg+deh+cfh)}{dfh}}{105d^2f^2h^2} + \frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2} + \frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} + \frac{2\int \frac{\frac{1}{2}d(35a^2d^2f^2h^2(3Adfh-C(deg+cfg+ceh))+28abCdfh(2d^2eg(fg+eh)+2c^2fh(fg+eh)+cd(2f^2g^2+3efgh+2e^2h^2))-b^2(35Ad}{}}{}$$

$$= \frac{2\Big(35Ab^2dfh+8a^2Cdfh-25b^2C(deg+cfg+ceh)-38abC(dfg+deh+cfh)+\frac{24b^2C(dfg+deh+cfh)}{dfh}}{105d^2f^2h^2} + \frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2} + \frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - \frac{(2(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8}{} + \frac{(35a^2d^2f^2h^2(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-14abdfh(15Ad^2f^2gh^2+C(4c^2fh^2(f}{}$$

$$= \frac{2\Big(35Ab^2dfh + 8a^2Cdfh - 25b^2C(deg + cfg + ceh) - 38abC(dfg + deh + cfh) + \frac{24b^2C(dfg+deh+cf}{dfh}}{105d^2f^2h^2}$$

$$+ \frac{4C(2adfh - 3b(dfg + deh + cfh))(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{35d^2f^2h^2}$$

$$+ \frac{2C(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{7dfh}$$

$$+ \frac{\Big((35a^2d^2f^2h^2(3Adfh^2 + cCh(fg - eh) + Cdg(2fg + eh)) - 14abdfh(15Ad^2f^2gh^2 + C(4c^2fh^2}{}$$

$$- \frac{\Big(2(35a^2Cd^2f^2h^2(dfg + deh + cfh) - 7abdfh(15Ad^2f^2h^2 + C(8c^2f^2h^2 + 7cdfh(fg + eh) + d^2(}{}$$

$$= \frac{2\Big(35Ab^2dfh + 8a^2Cdfh - 25b^2C(deg + cfg + ceh) - 38abC(dfg + deh + cfh) + \frac{24b^2C(dfg+deh+cf}{dfh}}{105d^2f^2h^2}$$

$$+ \frac{4C(2adfh - 3b(dfg + deh + cfh))(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{35d^2f^2h^2}$$

$$+ \frac{2C(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{7dfh}$$

$$- \frac{4\sqrt{-de + cf}(35a^2Cd^2f^2h^2(dfg + deh + cfh) - 7abdfh(15Ad^2f^2h^2 + C(8c^2f^2h^2 + 7cdfh(fg +}{}$$

$$+ \frac{\Big((35a^2d^2f^2h^2(3Adfh^2 + cCh(fg - eh) + Cdg(2fg + eh)) - 14abdfh(15Ad^2f^2gh^2 + C(4c^2fh^2}{}$$

$$= \frac{2\Big(35Ab^2dfh + 8a^2Cdfh - 25b^2C(deg + cfg + ceh) - 38abC(dfg + deh + cfh) + \frac{24b^2C(dfg+deh+cf}{dfh}}{105d^2f^2h^2}$$

$$+ \frac{4C(2adfh - 3b(dfg + deh + cfh))(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{35d^2f^2h^2}$$

$$+ \frac{2C(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}{7dfh}$$

$$- \frac{4\sqrt{-de + cf}(35a^2Cd^2f^2h^2(dfg + deh + cfh) - 7abdfh(15Ad^2f^2h^2 + C(8c^2f^2h^2 + 7cdfh(fg +}{}$$

$$+ \frac{2\sqrt{-de + cf}(35a^2d^2f^2h^2(3Adfh^2 + cCh(fg - eh) + Cdg(2fg + eh)) - 14abdfh(15Ad^2f^2gh^2 +}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.54 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\left(-2d^2\sqrt{-c+\frac{de}{f}}(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)\right.}{}$$

```
[In] Integrate[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x]),x]
```

```
[Out] (2*(-2*d^2*Sqrt[-c + (d*e)/f]*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h)
 - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h)
 + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*f*g
 + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h) + c*
d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e*f^2*
g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*(e + f*x)*(g + h*x) + d^2*Sqrt[-c + (
d*e)/f]*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(35*a^2*C*d^2*f^2*h^2 - 14*a*b*C*
d*f*h*(4*c*f*h + d*(4*f*g + 4*e*h - 3*f*h*x)) + b^2*(35*A*d^2*f^2*h^2 + C*(
24*c^2*f^2*h^2 + c*d*f*h*(23*f*g + 23*e*h - 18*f*h*x) + d^2*(24*e^2*h^2 + e
*f*h*(23*g - 18*h*x) + 3*f^2*(8*g^2 - 6*g*h*x + 5*h^2*x^2))))) - (2*I)*(d*e
 - c*f)*h*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h) - 7*a*b*d*f*h*(15*A
*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 +
7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h) +
2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h) + c*d^2*f*h*(10*f^2*g^2
+ 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e*f^2*g^2*h + 5*e^2*f*g*h^
2 + 6*e^3*h^3))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d
*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(35*a^2*d^2*f^2*h^2*(3*A*d*
f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - 14*a*b*d*f*h*(15*A*d^
2*e*f^2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*g*h +
 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f
^2*h^2*(c*f*(-(f*g) + e*h) + d*e*(f*g + 2*e*h)) + C*(24*c^3*f^3*h^2*(-(f*g)
 + e*h) + c^2*d*f^2*h*(-23*f^2*g^2 + 6*e*f*g*h + 17*e^2*h^2) + 2*c*d^2*f*(-
12*f^3*g^3 + 3*e*f^2*g^2*h + e^2*f*g*h^2 + 8*e^3*h^3) + d^3*e*(24*f^3*g^3 +
 17*e*f^2*g^2*h + 16*e^2*f*g*h^2 + 48*e^3*h^3))))*(c + d*x)^(3/2)*Sqrt[(d*(
e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcS
inh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(
105*d^5*Sqrt[-c + (d*e)/f]*f^4*h^4*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
])
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 1238, normalized size of antiderivative = 1.13

method	result	size
elliptic	Expression too large to display	1238
default	Expression too large to display	12279

```
[In] int((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/7*C*b^2/d/f/h*x^2*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)^(1/2)+2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*
g))/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*
x+c*e*g)^(1/2)+2/3*(b^2*A+C*a^2-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5/2*d*
e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2*c*f*h
+2*d*e*h+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c
*f*g*x+d*e*g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+
3*d*e*h+3*d*f*g))/d/f/h*c*e*g-2/3*(b^2*A+C*a^2-2/7*C*b^2/d/f/h*(5/2*c*e*h+5
/2*c*f*g+5/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))
/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g
/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h
+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g
*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))
^(1/2))+2*(2*a*b*A-4/7*C*b^2/d/f/h*c*e*g-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*
f*h+3*d*e*h+3*d*f*g))/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2/3*(b^2*A+C*a^
2-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/
f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(c*f*
h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1
/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e
*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))
^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/
2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1665, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^2*d^4*f^4*h^4*x^2 + 24*C*b^2*d^4*f^4*g^2*h^2 + (23*C*b^2*d
^4*e*f^3 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*f^4)*g*h^3 + (24*C*b^2*d^4*e^2*f
^2 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*e*f^3 + (24*C*b^2*c^2*d^2 - 56*C*a*b*c
*d^3 + 35*(C*a^2 + A*b^2)*d^4)*f^4)*h^4 - 6*(3*C*b^2*d^4*f^4*g*h^3 + (3*C*b
^2*d^4*e*f^3 + (3*C*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*h^4)*x)*sqrt(d*x + c)*sqr
t(f*x + e)*sqrt(h*x + g) + (48*C*b^2*d^4*f^4*g^4 + 16*(C*b^2*d^4*e*f^3 + (C
*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*g^3*h + (11*C*b^2*d^4*e^2*f^2 + 14*(C*b^2*c*
d^3 - 3*C*a*b*d^4)*e*f^3 + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*(C*a^2 +
 A*b^2)*d^4)*f^4)*g^2*h^2 + (16*C*b^2*d^4*e^3*f + 14*(C*b^2*c*d^3 - 3*C*a*b
*d^4)*e^2*f^2 + 7*(2*C*b^2*c^2*d^2 - 6*C*a*b*c*d^3 + 5*(C*a^2 + A*b^2)*d^4)
*e*f^3 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35*(C*a^2 + A
*b^2)*c*d^3)*f^4)*g*h^3 + (48*C*b^2*d^4*e^4 + 16*(C*b^2*c*d^3 - 7*C*a*b*d^4
)*e^3*f + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*(C*a^2 + A*b^2)*d^4)*e^2*
f^2 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35*(C*a^2 + A*b^
2)*c*d^3)*e*f^3 + (48*C*b^2*c^4 - 112*C*a*b*c^3*d - 210*A*a*b*c*d^3 + 315*A
*a^2*d^4 + 70*(C*a^2 + A*b^2)*c^2*d^2)*f^4)*h^4)*sqrt(d*f*h)*weierstrassPIn
verse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2
*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*
g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*
d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x +
 d*f*g + (d*e + c*f)*h)/(d*f*h)) + 6*(24*C*b^2*d^4*f^4*g^3*h + 4*(5*C*b^2*d
^4*e*f^3 + (5*C*b^2*c*d^3 - 14*C*a*b*d^4)*f^4)*g^2*h^2 + (20*C*b^2*d^4*e^2*
f^2 + (18*C*b^2*c*d^3 - 49*C*a*b*d^4)*e*f^3 + (20*C*b^2*c^2*d^2 - 49*C*a*b*
c*d^3 + 35*(C*a^2 + A*b^2)*d^4)*f^4)*g*h^3 + (24*C*b^2*d^4*e^3*f + 4*(5*C*b
^2*c*d^3 - 14*C*a*b*d^4)*e^2*f^2 + (20*C*b^2*c^2*d^2 - 49*C*a*b*c*d^3 + 35*
(C*a^2 + A*b^2)*d^4)*e*f^3 + (24*C*b^2*c^3*d - 56*C*a*b*c^2*d^2 - 105*A*a*b
*d^4 + 35*(C*a^2 + A*b^2)*c*d^3)*f^4)*h^4)*sqrt(d*f*h)*weierstrassZeta(4/3*
(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)
/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*
(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f
- 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d
^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(
d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*
e + c*f)*h)/(d*f*h))))/(d^5*f^5*h^5)
```

Sympy [F]

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\,(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**2*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2
),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*
x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Giac [F]

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,\left(a+b\,x\right)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1
/2)),x)
```

```
[Out] int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1
/2)), x)
```

3.27 $\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 611

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} - \frac{2\sqrt{-de+cf}(10aCdfh(dfg+deh+cfh)-b(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f^2g^2+\cdots}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}\cdots} + \frac{2\sqrt{-de+cf}(5adfh(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-eh\cdots}{15d\cdots}$$

```
[Out] 4/15*C*(a*d*f*h-2*b*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g
)^(1/2)/d^2/f^2/h^2+2/5*C*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)
/d/f/h-2/15*(10*a*C*d*f*h*(c*f*h+d*e*h+d*f*g)-b*(15*A*d^2*f^2*h^2+C*(8*c^2*
f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*Elliptic
E(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*
(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(5/2)/h^3/
(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/15*(5*a*d*f*h*(3*A*d*f*h^2+C*(
c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))-b*(15*A*d^2*f^2*g*h^2+C*(4*c^2*f*h^2*(-e*h
+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*g*h+8*f^2
*g^2))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-
c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-
c*h+d*g))^(1/2)/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1615, 1629, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(5adfh(3Adfh^2+cCh(fg-eh)+C\ldots\right.}{15d^3f^{5/\ldots}}$$
$$+\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(-10aCdfh(cfh+deh+dfg)+15Abd^2f^2h\ldots\right.}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{4C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(adfh-2b(cfh+deh+dfg))}{15d^2f^2h^2}$$
$$+\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

```
[In] Int[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (4*C*(a*d*f*h - 2*b*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sq
rt[g + h*x])/(15*d^2*f^2*h^2) + (2*C*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x])/(5*d*f*h) + (2*Sqrt[-(d*e) + c*f]*(15*A*b*d^2*f^2*h^2 - 10*a
*C*d*f*h*(d*f*g + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*
h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*
f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(15*d^3*f^(5/2)*h^3*Sqrt[e + f*x]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(5*a*d*f*h*(3*A*d*f*
h^2 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - b*(15*A*d^2*f^2*g*h^2 + C*
(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*
(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(15*d^3*f^(5/2)*h^3*Sqrt[e
 + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1615

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f*
g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{\int \frac{-2bcCeg+5aAdfh-aC(deg+cfg+ceh)+(5Abdfh-3bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2C(adfh-2b(dfg+deh+cfh))x^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{5dfh}$$

$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{2\int \frac{\frac{1}{2}d\left(5adfh(3Adfh-C(deg+cfg+ceh))+2bC\left(2d^2eg(fg+eh)+2c^2fh(fg+eh)+cd\left(2f^2g^2+3efgh+2e^2h^2\right)\right)\right)+\frac{1}{2}d\left(15Abd^2f^2h^2-1\right.}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{15d^3f^2h^2}$$

$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} + \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} + \frac{(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f^2g^2+7efg}{15d^2f^2h^3} + \frac{(5adfh(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-eh)+cd}{15d^2f^2h^3}$$

$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{\Big((5adfh(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-eh)+}{15d^2}$$
$$+ \frac{\Big((15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f^2g^2+7e}{15d^2f^2h^3\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g}{dg}}}$$
$$+ \frac{\Big((5adfh(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-eh)+}{15}$$

$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g}{dg}}}$$
$$+ \frac{2\sqrt{-de+cf}(5adfh(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b(15Ad^2f^2gh^2+C(4c^2fh^2(}{1}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.26 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}\left(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC\left(8c^2f^2h^2+7cdfh(fg+eh)+d^2\left(8f^2g\right.\right.\right.\right.}{}$$

```
[In] Integrate[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
]),x]
```

```
[Out] (-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(d*f*g + d
*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2
 + 7*e*f*g*h + 8*e^2*h^2)))*(e + f*x)*(g + h*x)) + C*d^2*Sqrt[-c + (d*e)/f]
*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(4*b*c*f*h - 5*a*d*f*h + b*d*(4*f*g + 4*
e*h - 3*f*h*x)) - I*(d*e - c*f)*h*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(d*f*g
 + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2
*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c +
d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e
)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(5*a*d*f*h*(3
*A*d*f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - b*(15*A*d^2*e*f^
2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*g*h + 3*e^2
*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(
d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*A
rcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])
)/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2Cbx\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(Ca-\frac{2Cb(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh\,x^3+cfh\,x^2}}{3dfh}\right.}{}$
default	Expression too large to display

```
[In] int((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/5*C*b/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*
(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/
2)+2*(A*a-2/5*C*b/d/f/h*c*e*g-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+2*d*f
*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(
1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h
*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x
+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*(A*b-2/5*C*b/d/f/h*
(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+2*d
*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c
/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h
*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE((
(x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g
/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1068, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{2\left(3\left(3\,Cbd^3f^3h^3x-4\,Cbd^3f^3gh^2-\left(4\,Cbd^3ef^2+\left(4\,Cbcd^2-5\,Cad^3\right)f^3\right)h^3\right)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}-\right.}{}$$

```
[In] integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 2/45*(3*(3*C*b*d^3*f^3*h^3*x - 4*C*b*d^3*f^3*g*h^2 - (4*C*b*d^3*e*f^2 + (4*
C*b*c*d^2 - 5*C*a*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
- (8*C*b*d^3*f^3*g^3 + (3*C*b*d^3*e*f^2 + (3*C*b*c*d^2 - 10*C*a*d^3)*f^3)*g
^2*h + (3*C*b*d^3*e^2*f + (3*C*b*c*d^2 - 5*C*a*d^3)*e*f^2 + (3*C*b*c^2*d -
5*C*a*c*d^2 + 15*A*b*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e^3 + (3*C*b*c*d^2 - 10*C
*a*d^3)*e^2*f + (3*C*b*c^2*d - 5*C*a*c*d^2 + 15*A*b*d^3)*e*f^2 + (8*C*b*c^3
 - 10*C*a*c^2*d + 15*A*b*c*d^2 - 45*A*a*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstr
assPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f
 + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2
*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3
- 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f
*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) - 3*(8*C*b*d^3*f^3*g^2*h + (7*C*b*d^
3*e*f^2 + (7*C*b*c*d^2 - 10*C*a*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e^2*f + (7*C*b
*c*d^2 - 10*C*a*d^3)*e*f^2 + (8*C*b*c^2*d - 10*C*a*c*d^2 + 15*A*b*d^3)*f^3)
*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*
h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3
- 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^
3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^
3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h
+ (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 -
3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)
*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*
f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))))/(d^4*f^4*h^4)
```

Sympy [F]

$$\int \frac{(a+bx)\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\,(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x
)
```

```
[Out] Integral((A + C*x**2)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x))
, x)
```

Maxima [F]

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Giac [F]

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,(a+b\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2
)),x)
```

```
[Out] int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2
)), x)
```

3.28 $\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 368

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$-\frac{4C\sqrt{-de+cf}(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right.\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2/3*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-4/3*C*(c*f*h+d*e*h+d*
f*g)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^2
/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(3*A*d*f*h^2+C*
(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)
^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+
d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x
+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1629, 164, 115, 114, 122, 121}

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

```
[In] Int[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) - (4*C*Sqrt[-(d*e
) + c*f]*(d*f*g + d*e*h + c*f*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h
*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)*h^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x)
)/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) +
C*d*g*(2*f*g + e*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d
*e - c*f)*h)/(f*(d*g - c*h))])/(3*d^2*f^(3/2)*h^2*Sqrt[e + f*x]*Sqrt[g + h*
x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{2\int \frac{\frac{1}{2}d(3Adfh-C(deg+cfg+ceh))-Cd(dfg+deh+cfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3d^2fh}$$

$$= \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{(2C(dfg+deh+cfh))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{3dfh^2} + \frac{(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh^2}$$

$$= \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{\left((3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{3dfh^2\sqrt{e+fx}} - \frac{\left(2C(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{3dfh^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{4C\sqrt{-de+cf}(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{\left((3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\cdots}}}{3dfh^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{4C\sqrt{-de+cf}(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{2\sqrt{-de+cf}(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\cdots\right.\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.77 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$= \frac{\sqrt{c+dx}\left(2Cd^2fh(e+fx)(g+hx) - \frac{4Cd^2(dfg+deh+cfh)(e+fx)(g+hx)}{c+dx} - 4iC\sqrt{-c+\frac{de}{f}}fh(dfg+deh+cfh)\cdots\right.}{\cdots}$$

```
[In] Integrate[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(2*C*d^2*f*h*(e + f*x)*(g + h*x) - (4*C*d^2*(d*f*g + d*e*h +
 c*f*h)*(e + f*x)*(g + h*x))/(c + d*x) - (4*I)*C*Sqrt[-c + (d*e)/f]*f*h*(d*
f*g + d*e*h + c*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(
d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
 d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + ((2*I)*d*h*(3*A*d*f^2*h + c*C*f*
(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(
c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c +
(d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/
f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\cdots}}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+\cdots}}\right.}{}$
default	Expression too large to display

```
[In] int((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/3*C/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*
x+c*e*g)^(1/2)+2*(A-2/3*C/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/f)*
((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(
1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g
)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-
4/3*C/d/f/h*(c*f*h+d*e*h+d*f*g)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d
)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x
^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x
+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h
)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.11

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}Cd^2f^2h^2 + (2\,Cd^2f^2g^2 + (Cd^2ef + Ccdf^2)gh + (2\,Cd^2e^2 + Ccdef + (2\,Cc\right.}{}$$

```
[In] integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*d^2*f^2*h^2 + (2*C*d^2*f
^2*g^2 + (C*d^2*e*f + C*c*d*f^2)*g*h + (2*C*d^2*e^2 + C*c*d*e*f + (2*C*c^2
+ 9*A*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^
2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4
/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d
^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 +
2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f
*h)) + 6*(C*d^2*f^2*g*h + (C*d^2*e*f + C*c*d*f^2)*h^2)*sqrt(d*f*h)*weierstr
assZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c
^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3
)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*
c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPIn
verse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2
*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*
g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*
d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x +
 d*f*g + (d*e + c*f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((A + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
[Out] integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
[Out] integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.29 $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 465

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2C\sqrt{-de+cf}(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A+\frac{a^2C}{b^2}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*C*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/b/d/
h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*C*(a*h+b*g)*Elliptic
F(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*
(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b
^2/d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A+a^2*C/b^2)*EllipticPi(f^(1/
2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f
/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)
/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)} - \frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} + \frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

```
[In] Int[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(b*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]) - (2*C*Sqrt[-(d*e) + c*f]*(b*g + a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt
[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*d*Sqrt[f]*h*Sqrt[e
+ f*x]*Sqrt[g + h*x]) - (2*(A + (a^2*C)/b^2)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e -
 c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]]
, ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt
[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(A+\frac{a^2C}{b^2}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&\quad +\int \frac{-\frac{aC}{b^2}+\frac{Cx}{b}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&= -\left(\left(2\left(A+\frac{a^2C}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx,x,\sqrt{c+dx}\right)\right) \\
&\quad +\frac{C\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{bh}-\frac{(C(bg+ah))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2h} \\
&= -\frac{\left(2\left(A+\frac{a^2C}{b^2}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx,x,\sqrt{c+dx}\right)}{\sqrt{e+fx}} \\
&\quad -\frac{\left(C(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{b^2h\sqrt{e+fx}} \\
&\quad +\frac{\left(C\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{bh\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}
\end{aligned}$$

$$= \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{\left(2\left(A+\frac{a^2C}{b^2}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int\frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}dx,x,\sqrt{c+dx}\right)}{\sqrt{e+fx}\sqrt{g+hx}} - \frac{\left(C(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2h\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2C\sqrt{-de+cf}(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A+\frac{a^2C}{b^2}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.55 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.23

$$\int\frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx = -\frac{2\left(b^2cCd^2e\sqrt{-c+\frac{de}{f}}g-abCd^3e\sqrt{-c+\frac{de}{f}}g-b^2c^2Cd\sqrt{-c+\frac{de}{f}}fg+abcCd^2\sqrt{-c+\frac{de}{f}}fg-b^2c^2Cde\right.}{}$$

```
[In] Integrate[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] (-2*(b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g - a*b*C*d^3*e*Sqrt[-c + (d*e)/f]*g
- b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g + a*b*c*C*d^2*Sqrt[-c + (d*e)/f]*f*g -
 b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h + a*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h +
b^2*c^3*C*Sqrt[-c + (d*e)/f]*f*h - a*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h + b^2
*c*C*d*Sqrt[-c + (d*e)/f]*f*g*(c + d*x) - a*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g*
(c + d*x) + b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h*(c + d*x) - a*b*C*d^2*e*Sqrt[-
c + (d*e)/f]*h*(c + d*x) - 2*b^2*c^2*C*Sqrt[-c + (d*e)/f]*f*h*(c + d*x) + 2
```

```
*a*b*c*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x) + b^2*c*C*Sqrt[-c + (d*e)/f]*f*
h*(c + d*x)^2 - a*b*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x)^2 + I*b*C*(b*c - a
*d)*(d*e - c*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d
*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*b*d*(b*c*C*e - a*C*d*e + a*c*C*
f + A*b*d*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g
 + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x
]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*A*b^2*d^2*f*h*(c + d*x)^(3/2)*Sqrt
[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[
-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d
*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*a^2*C*d^2*f*h*(c + d*x)^(3/2)*Sq
rt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticP
i[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
 d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(b^2*d^2*(-(b*c) + a*d)*Sqrt[-c
+ (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2Ca\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2C\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\left(\right.}{b\sqrt{dfh\,x^3+cfh\,x^2\cdots}}\right.}{\sqrt{dx+\cdots}}$
default	Expression too large to display

```
[In] int((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
-2*C*a/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*(
(x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+
c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f
)/(-g/h+c/d))^(1/2))+2*C/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g
/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*
f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)
/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/
h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))+2*(A*b^2+C*a^2)/b^3*(g/h-e/f)
*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^
(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*
```

```
g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a
/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*x**2+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x
)
[Out] Integral((A + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x))
, x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="maxima")
[Out] integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2))
,x)
```

```
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2))
, x)
```

3.30 $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 738

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{\left(Ab+\frac{a^2C}{b}\right)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{\sqrt{-de+cf}(a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right.\right.}{b^2d(bc-ad)\sqrt{f}(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Ade}{b^2(bc-ad)^2\sqrt{f}(b}$$

```
[Out] -(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b
*e)/(-a*h+b*g)/(b*x+a)+(A*b+a^2*C/b)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d
*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*
x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(f*x+
e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+(a^2*C*d*f-2*a*b*C*(c*f+d*e)+b^2*(-A*
d*f+2*C*c*e))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h
/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+
g)/(-c*h+d*g))^(1/2)/b^2/d/(-a*d+b*c)/(-a*f+b*e)/f^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2)-(a^4*C*d*f*h-A*b^4*(c*e*h+c*f*g+d*e*g)-2*a^3*b*C*(c*f*h+d*e*h+d*f
*g)-2*a*b^3*(-A*c*f*h-A*d*e*h-A*d*f*g+2*C*c*e*g)-3*a^2*b^2*(A*d*f*h-C*(c*e*
h+c*f*g+d*e*g)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+
d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*
x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+
b*e)/(-a*h+b*g)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1619, 1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-}{f(dg}\right.}{b^2d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)(be-af)}$$
$$+\frac{\sqrt{f}\sqrt{g+hx}\left(\frac{a^2C}{b}+Ab\right)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{e+fx}(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$
$$-\frac{\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))+}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+}}$$

```
[In] Int[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] -(((A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*(a + b*x))) + ((A*b + (a^2*C)/b)*Sqrt[f]*Sqrt[-(d*e
) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sq
rt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])
/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g
 - c*h)]) + (Sqrt[-(d*e) + c*f]*(a^2*C*d*f - 2*a*b*C*(d*e + c*f) + b^2*(2*c
*C*e - A*d*f))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*
h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/(b^2*d*(b*c - a*d)*Sqrt[f]*(b*e - a*f)*Sqrt[e + f*
x]*Sqrt[g + h*x]) - (Sqrt[-(d*e) + c*f]*(a^4*C*d*f*h - A*b^4*(d*e*g + c*f*g
 + c*e*h) - 2*a^3*b*C*(d*f*g + d*e*h + c*f*h) - 2*a*b^3*(2*c*C*e*g - A*d*f*
g - A*d*e*h - A*c*f*h) - 3*a^2*b^2*(A*d*f*h - C*(d*e*g + c*f*g + c*e*h)))*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-
((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*
e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*(b*c - a*d)^2*Sqrt[f]*(b
*e - a*f)*(b*g - a*h)*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
```

```
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1619

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(
d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(d
*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(m
 + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g
*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2
 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && In
tegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\int \frac{-Ab^2(deg+cfg+ceh)-2ab(cCeg-Adfg-Adeh-Acfh)-a^2(2Adfh-C(deg+cfg+ceh))+2\left(b^2cCeg+a^2C(dfg+deh+cfh)+ab(Adfh-C(deg}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$+\frac{\int \frac{2bcCeg-2aCdeg-2acCfg+\frac{2a^2Cdfg}{b}-2acCeh+\frac{2a^2Cdeh}{b}+\frac{2a^2cCfh}{b}+aAdfh-\frac{a^3Cdfh}{b^2}+\left(Abdfh+\frac{a^2Cdfh}{b}\right)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2(bc-ad)(be-af)(bg-ah)}$$

$$+\frac{(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Adeh-}{2b^2(bc-ad)(be-af)(bg-}$$

$$= -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$+\frac{(a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b^2(bc-ad)(be-af)}$$

$$+\frac{\left(\left(Ab+\frac{a^2C}{b}\right)df\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2(bc-ad)(be-af)(bg-ah)}$$

$$-\frac{(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Adeh-}{b^2(bc-ad}$$

$$= -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$

$$+\frac{\left((a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{2b^2(bc-ad)(be-af)\sqrt{e+fx}}$$

$$-\frac{\Big((a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Adeh}{b^2(bc-}$$

$$+\frac{\left(\left(Ab+\frac{a^2C}{b}\right)df\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{2(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\left(Ab+\frac{a^2C}{b}\right)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{\left((a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{d}{dg}}}}{2b^2(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}} - \frac{\left((a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Adeh-\right.}{b^2(bc-a}$$

$$= -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)} + \frac{\left(Ab+\frac{a^2C}{b}\right)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{\sqrt{-de+cf}(a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\right.}{b^2d(bc-ad)\sqrt{f}(be-af)\sqrt{e+fx}\sqrt{g+hx}} - \frac{\sqrt{-de+cf}(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg}{b^2(bc-ad)^2\sqrt{f}(}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.65 (sec) , antiderivative size = 3935, normalized size of antiderivative = 5.33

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
]),x]
```

```
[Out] ((-(A*b^2) - a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)
*(b*e - a*f)*(b*g - a*h)*(a + b*x)) - ((c + d*x)^(3/2)*(A*b^4*c*Sqrt[-c + (
d*e)/f]*f*h + a^2*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*A*b^3*d*Sqrt[-c + (d*e
)/f]*f*h - a^3*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^4*c*d^2*e*Sqrt[-c + (d*e
)/f]*g)/(c + d*x)^2 + (a^2*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2
- (a*A*b^3*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (a^3*b*C*d^3*e*Sqrt[-c
 + (d*e)/f]*g)/(c + d*x)^2 - (A*b^4*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)
```

```
^2 - (a^2*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^3*c*d^2*
Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a^3*b*c*C*d^2*Sqrt[-c + (d*e)/f]*f*g
)/(c + d*x)^2 - (A*b^4*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 - (a^2*b^2
*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^3*c*d^2*e*Sqrt[-c + (
d*e)/f]*h)/(c + d*x)^2 + (a^3*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2
 + (A*b^4*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (a^2*b^2*c^3*C*Sqrt[-c
+ (d*e)/f]*f*h)/(c + d*x)^2 - (a*A*b^3*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d
*x)^2 - (a^3*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (A*b^4*c*d*Sqr
t[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b^2*c*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c
+ d*x) - (a*A*b^3*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a^3*b*C*d^2*Sqrt
[-c + (d*e)/f]*f*g)/(c + d*x) + (A*b^4*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x
) + (a^2*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*A*b^3*d^2*e*Sqrt[
-c + (d*e)/f]*h)/(c + d*x) - (a^3*b*C*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)
 - (2*A*b^4*c^2*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) - (2*a^2*b^2*c^2*C*Sqrt[-
c + (d*e)/f]*f*h)/(c + d*x) + (2*a*A*b^3*c*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d
*x) + (2*a^3*b*c*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x) + (I*b*(A*b^2 + a^2*
C)*(-(b*c) + a*d)*(-(d*e) + c*f)*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x
))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c
 + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x]
 + (I*b*(2*a*b*(c^2*C + A*d^2)*f + a^2*C*d*(d*e - c*f) - b^2*(2*c^2*C*e + A
*d^2*e + A*c*d*f))*(-(b*g) + a*h)*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x)
)]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c
+ (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x]
- ((4*I)*a*b^3*c*C*d*e*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1
 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e -
 b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e
*h - c*f*h)])/Sqrt[c + d*x] - (I*A*b^4*d^2*e*g*Sqrt[1 - c/(c + d*x) + (d*e)
/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b
*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]]
, (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*a^2*b^2*C*d^2*e*
g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/
(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqr
t[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c +
d*x] - (I*A*b^4*c*d*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1
- c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e -
b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*
h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*a^2*b^2*c*C*d*f*g*Sqrt[1 - c/(c + d*x)
+ (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*Elliptic
Pi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c
+ d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((2*I)*a*A*b^3*d
^2*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (
d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSin
h[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt
[c + d*x] - ((2*I)*a^3*b*C*d^2*f*g*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x
))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f
```

```
)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - (I*A*b^4*c*d*e*h*Sqrt[1 - c/(c + d*
x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*Ellip
ticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt
[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*a^2*b^
2*c*C*d*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x
) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*A
rcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])
/Sqrt[c + d*x] + ((2*I)*a*A*b^3*d^2*e*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c
+ d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f -
a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*
g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((2*I)*a^3*b*C*d^2*e*h*Sqrt[1
- c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d
*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d
*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((
2*I)*a*A*b^3*c*d*f*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c
/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c
*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h -
 c*f*h)])/Sqrt[c + d*x] - ((2*I)*a^3*b*c*C*d*f*h*Sqrt[1 - c/(c + d*x) + (d*
e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-(
(b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x
]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((3*I)*a^2*A*b^2*d^2*
f*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g
)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[S
qrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c
+ d*x] + (I*a^4*C*d^2*f*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[
1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e
- b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*
e*h - c*f*h)])/Sqrt[c + d*x]))/(b^2*d*(b*c - a*d)*(-(b*c) + a*d)*Sqrt[-c +
(d*e)/f]*(-(b*e) + a*f)*(-(b*g) + a*h)*Sqrt[e + ((c + d*x)*(f - (c*f)/(c +
d*x)))/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 1269, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	17416

```
[In] int((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
1/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*
```

```
b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2
+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)+2*(C/b^2-1/2*a/b^2*d*f*h*(A*b
^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*
c*f*g+a*b^2*d*e*g-b^3*c*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/
(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2
+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f
))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-d*f*h*(A*b^2+C*a^2)/(a^3*d*f*h-a^2*
b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e
*g)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g
*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-
g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+
e/f)/(-g/h+c/d))^(1/2)))+(3*A*a^2*b^2*d*f*h-2*A*a*b^3*c*f*h-2*A*a*b^3*d*e*h
-2*A*a*b^3*d*f*g+A*b^4*c*e*h+A*b^4*c*f*g+A*b^4*d*e*g-C*a^4*d*f*h+2*C*a^3*b*
c*f*h+2*C*a^3*b*d*e*h+2*C*a^3*b*d*f*g-3*C*a^2*b^2*c*e*h-3*C*a^2*b^2*c*f*g-3
*C*a^2*b^2*d*e*g+4*C*a*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*
d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^3*(g/h-e/f)*((x+g/h)
/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*
f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/
(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/
h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Cx^2}{(a + bx)^2 \sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} \, dx = \int \frac{Cx^2 + A}{(bx + a)^2 \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}} \, dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Giac [F]

$$\int \frac{A + Cx^2}{(a + bx)^2 \sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} \, dx = \int \frac{Cx^2 + A}{(bx + a)^2 \sqrt{dx + c} \sqrt{fx + e} \sqrt{hx + g}} \, dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{(a + bx)^2 \sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} \, dx = \int \frac{C\,x^2 + A}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,{(a + b\,x)}^2\,\sqrt{c + d\,x}} \, dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2
)),x)
```

```
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2
)), x)
```

3.31 $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 1395

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(C(3adfh-5b(dfg+deh+cfh))(adfh-3b(dfg+deh+cfh))+8bdf}{24}$$
$$+\frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$
$$+\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(C(3adfh-5b(dfg+deh+cfh))(adfh-3b(dfg+deh+cfh))+8bdfh(3Abdfh-C}{24bd^3f^3h^3\sqrt{}}$$
$$+\frac{(be-af)\sqrt{bg-ah}(3a^2Cd^2f^2h^2+6abCdfh(cfh+2d(fg+eh))-b^2(24Ad^2f^2h^2+C(5c^2f^2h^2+4cdfh(f}{24b^2d^2f^3h^3\sqrt{fg}}$$
$$-\frac{\sqrt{-dg+ch}(4bdfh(C(b(deg+cfg+ceh)+a(dfg+deh+cfh))(3adfh-5b(dfg+deh+cfh))+2dfh(}{}$$

```
[Out] -1/24*(4*b*d*f*h*(C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))*(3*a*d*f*
h-5*b*(c*f*h+d*e*h+d*f*g))+2*d*f*h*(3*b^2*c*C*e*g+2*a^2*C*(c*f*h+d*e*h+d*f*
g)-a*b*(12*A*d*f*h-5*C*(c*e*h+c*f*g+d*e*g))))+(a*d*f*h+b*(c*f*h+d*e*h+d*f*g
))*(C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))
+8*b*d*f*h*(3*A*b*d*f*h-C*(2*b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))
)*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)
^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f
*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*(
(-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/d^3/f^3/h^4/(-a*d+b*c)^(1/
```

```
2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/24*(C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*g))*
(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))+8*b*d*f*h*(3*A*b*d*f*h-C*(2*b*(c*e*h+c*f*
g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)
/b/d^2/f^3/h^3/(d*x+c)^(1/2)+1/3*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2
)*(h*x+g)^(1/2)/d/f/h+1/12*C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(1
/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+1/24*(-a*f+b*e)*(
3*a^2*C*d^2*f^2*h^2+6*a*b*C*d*f*h*(c*f*h+2*d*(e*h+f*g))-b^2*(24*A*d^2*f^2*h
^2+C*(5*c^2*f^2*h^2+4*c*d*f*h*(e*h+f*g)+d^2*(15*e^2*h^2+14*e*f*g*h+15*f^2*g
^2))))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1
/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*
((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b^2/d^2/f^3/h^3
/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1
/2)-1/24*(C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h-3*b*(c*f*h+d*e*h+d
*f*g))+8*b*d*f*h*(3*A*b*d*f*h-C*(2*b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f
*g))))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1
/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(
-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2
)/b/d^3/f^3/h^3/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 3.98 (sec) , antiderivative size = 1376, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1615, 1614, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a+bx)^{3/2}}{3dfh}$$

$$-\frac{\sqrt{ch-dg}((adfh+b(dfg+deh+cfh))(24Ad^2f^2h^2b^2+15C(dfg+deh+cfh)^2b^2-16Cdfh(deg+cfg+ \cdots}{\cdots}$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(24Ad^2f^2h^2b^2+15C(dfg+deh+cfh)^2b^2-16Cdfh(deg+cfg+ceh)b^2-22aCdfh(\cdots}{24bd^3f^3h^3\sqrt{\frac{(de-cf)(\cdots}{(be-af)(\cdots}}}$$

$$+\frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\sqrt{a+bx}}{12d^2f^2h^2}$$

$$+\frac{\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)\sqrt{e+f \cdots}}{24d^2f^2h^2\sqrt{c+dx}}$$

$$+\frac{(be-af)\sqrt{bg-ah}(-((24Ad^2f^2h^2+C((15f^2g^2+14efhg+15e^2h^2)d^2+4cfh(fg+eh)d+5c^2f^2h^2))b^2) \cdots}{24b^2d^2f^3h^3\sqrt{fg \cdots}}$$

```
[In] Int[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x
]),x]
```

```
[Out] ((24*A*b*d^2*f*h + (3*a^2*C*d^2*f*h)/b - 16*b*C*d*(d*e*g + c*f*g + c*e*h) -
 22*a*C*d*(d*f*g + d*e*h + c*f*h) + (15*b*C*(d*f*g + d*e*h + c*f*h)^2)/(f*h
))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(24*d^2*f^2*h^2*Sqrt[c + d*x]
) + (C*(3*a*d*f*h - 5*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x])/(12*d^2*f^2*h^2) + (C*(a + b*x)^(3/2)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) - (Sqrt[d*g - c*h]*Sqrt[f*g
- e*h]*(24*A*b^2*d^2*f^2*h^2 + 3*a^2*C*d^2*f^2*h^2 - 16*b^2*C*d*f*h*(d*e*g
+ c*f*g + c*e*h) - 22*a*b*C*d*f*h*(d*f*g + d*e*h + c*f*h) + 15*b^2*C*(d*f*g
 + d*e*h + c*f*h)^2)*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e
*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g
 - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h)
)])/(24*b*d^3*f^3*h^3*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]
*Sqrt[g + h*x]) + ((b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*C*d^2*f^2*h^2 + 6*a*b
*C*d*f*h*(c*f*h + 2*d*(f*g + e*h)) - b^2*(24*A*d^2*f^2*h^2 + C*(5*c^2*f^2*h
^2 + 4*c*d*f*h*(f*g + e*h) + d^2*(15*f^2*g^2 + 14*e*f*g*h + 15*e^2*h^2))))*
Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Ellipti
cF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])],
 -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(24*b^2*d^2*f^3*h
^3*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))]) - (Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h)
)*(24*A*b^2*d^2*f^2*h^2 + 3*a^2*C*d^2*f^2*h^2 - 16*b^2*C*d*f*h*(d*e*g + c*f
*g + c*e*h) - 22*a*b*C*d*f*h*(d*f*g + d*e*h + c*f*h) + 15*b^2*C*(d*f*g + d*
e*h + c*f*h)^2) + 4*b*d*f*h*(C*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*
h + c*f*h))*(3*a*d*f*h - 5*b*(d*f*g + d*e*h + c*f*h)) + 2*d*f*h*(3*b^2*c*C*
e*g + 2*a^2*C*(d*f*g + d*e*h + c*f*h) - a*b*(12*A*d*f*h - 5*C*(d*e*g + c*f*
g + c*e*h)))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x
))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(
d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[
-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g
- e*h))])/(24*b^2*d^3*Sqrt[b*c - a*d]*f^3*h^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
```

```
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f*
g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} + \frac{\int \frac{\sqrt{a+bx}(-3bcCeg+6aAdfh-aC(deg+cfg+ceh)+2(3Abdfh-2bC(deg+cfg+ceh)-aC(dfg+deh+cfh))x+C(3adfh-5b(dfg+deh+cfh))x^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh}$$

$$= \frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+ \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$+ \frac{\int \frac{-4adfh(3bcCeg-6aAdfh+aC(deg+cfg+ceh))-C(bceg+a(deg+cfg+ceh))(3adfh-5b(dfg+deh+cfh))-2\left(C(b(deg+cfg+ceh)+\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(24Abd^2fh+\frac{3a^2Cd^2fh}{b}-16bCd(deg+cfg+ceh)-22aCd(dfg+deh+cfh)+\frac{15bC(dfg+deh+cfh)^2}{fh}\cdots\right.}{24d^2f^2h^2\sqrt{c+dx}\cdots}$$

$$+ \frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+ \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$+ \frac{\int \frac{-\left((bdeg+acfh)\left(24Ab^2d^2f^2h^2+3a^2Cd^2f^2h^2-16b^2Cdfh(deg+cfg+ceh)-22abCdfh(dfg+deh+cfh)+15b^2C(dfg+deh+cfh)^2\right)\right)\cdots}{\cdots}}{\cdots}$$

$$+ \frac{((de-cf)(dg-ch)\left(24Ab^2d^2f^2h^2+3a^2Cd^2f^2h^2-16b^2Cdfh(deg+cfg+ceh)-22abCdfh(df\cdots\right.}{48bd^3f^3h^3\cdots}$$

$$= \frac{\left(24Abd^2fh+\frac{3a^2Cd^2fh}{b}-16bCd(deg+cfg+ceh)-22aCd(dfg+deh+cfh)+\frac{15bC(dfg+deh+cfh)^2}{fh}\cdots\right.}{24d^2f^2h^2\sqrt{c+dx}\cdots}$$

$$+ \frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+ \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$+ \frac{((be-af)(bg-ah)\left(3a^2Cd^2f^2h^2+6abCdfh(cfh+2d(fg+eh))-b^2(24Ad^2f^2h^2+C(5c^2f^2h^2\cdots\right.}{48b^2d^2f^3h^3\cdots}$$

$$- \frac{((adfh+b(dfg+deh+cfh))\left(24Ab^2d^2f^2h^2+3a^2Cd^2f^2h^2-16b^2Cdfh(deg+cfg+ceh)-22a\cdots\right.}{\cdots}$$

$$- \frac{\left(\left((dg-ch)\left(24Ab^2d^2f^2h^2+3a^2Cd^2f^2h^2-16b^2Cdfh(deg+cfg+ceh)-22abCdfh(dfg+deh+\cdots\right.\right.\right.}{24bd^3f^3h^3\sqrt{\frac{(d\cdots}{(be\cdots}}}$$

$$= \frac{\left(24Abd^2fh + \frac{3a^2Cd^2fh}{b} - 16bCd(deg + cfg + ceh) - 22aCd(dfg + deh + cfh) + \frac{15bC(dfg+deh+cfh)}{fh}\right.}{24d^2f^2h^2\sqrt{c+dx}}$$

$$+ \frac{C(3adfh - 5b(dfg + deh + cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+ \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$- \frac{\sqrt{dg-ch}\sqrt{fg-eh}(24Ab^2d^2f^2h^2 + 3a^2Cd^2f^2h^2 - 16b^2Cdfh(deg + cfg + ceh) - 22abCdfh(d}{24bd^3f^3h}$$

$$- \frac{\left(((adfh + b(dfg + deh + cfh))\left(24Ab^2d^2f^2h^2 + 3a^2Cd^2f^2h^2 - 16b^2Cdfh(deg + cfg + ceh) - \right.\right.}{}$$

$$+ \frac{\left((be - af)(bg - ah)\left(3a^2Cd^2f^2h^2 + 6abCdfh(cfh + 2d(fg + eh)) - b^2(24Ad^2f^2h^2 + C(5c^2f^2\right.\right.}{24b^2d}$$

$$= \frac{\left(24Abd^2fh + \frac{3a^2Cd^2fh}{b} - 16bCd(deg + cfg + ceh) - 22aCd(dfg + deh + cfh) + \frac{15bC(dfg+deh+cfh)}{fh}\right.}{24d^2f^2h^2\sqrt{c+dx}}$$

$$+ \frac{C(3adfh - 5b(dfg + deh + cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+ \frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$- \frac{\sqrt{dg-ch}\sqrt{fg-eh}(24Ab^2d^2f^2h^2 + 3a^2Cd^2f^2h^2 - 16b^2Cdfh(deg + cfg + ceh) - 22abCdfh(d}{24bd^3f^3h}$$

$$+ \frac{(be - af)\sqrt{bg-ah}(3a^2Cd^2f^2h^2 + 6abCdfh(cfh + 2d(fg + eh)) - b^2(24Ad^2f^2h^2 + C(5c^2f^2h^2}{24b^2d^2f^3h^3}$$

$$- \frac{\sqrt{-dg+ch}((adfh + b(dfg + deh + cfh))\left(24Ab^2d^2f^2h^2 + 3a^2Cd^2f^2h^2 - 16b^2Cdfh(deg + cfg\right.}{}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 39032 vs. 2(1395) = 2790.

Time = 40.08 (sec) , antiderivative size = 39032, normalized size of antiderivative = 27.98

$$\int \frac{(a+bx)^{3/2}\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [A] (verified)

Time = 6.74 (sec) , antiderivative size = 2228, normalized size of antiderivative = 1.60

method	result	size
elliptic	Expression too large to display	2228
default	Expression too large to display	92114

```
[In] int((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(1/3*C*b/d/f/h*x*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+
b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c
*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)
+1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b*d*f*
g))/b/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*
c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e
*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*A-1/3*C*b/d/f/h*a*
c*e*g-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b
*d*f*g))/b/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g/h-a/
b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e
/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)
/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*Elli
pticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-
a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b*A-1/3*C*b/d/f/h*(3/2*a*c*e*h+3/2*a*c*f
*g+3/2*a*d*e*g+3/2*b*c*e*g)-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c
*f*h+5/2*b*d*e*h+5/2*b*d*f*g))/b/d/f/h*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c
*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g
/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/
```

```
f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(
1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticP
i(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f
-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(b^2*A+C*a^2-1/3*C*b/d/f/h*(
2*a*c*f*h+2*a*d*e*h+2*a*d*f*g+2*b*c*e*h+2*b*c*f*g+2*b*d*e*g)-1/2*(2*C*a*b-1
/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b*d*f*g))/b/d/f/h*(3/
2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g
/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)
*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^
(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)
/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(
a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*
(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^
(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\left(A+Cx^2\right)(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**
(1/2),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
+ h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
 + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
 + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\left(C\,x^2+A\right)\,{\left(a+b\,x\right)}^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x
)^(1/2)),x)
```

```
[Out] int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x
)^(1/2)), x)
```

3.32 $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 937

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}}$$
$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{C\sqrt{dg-ch}\sqrt{fg-eh}(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)\right|}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+ \frac{C(be-af)\sqrt{bg-ah}(adfh+b(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{}}{\sqrt{fg-eh}\sqrt{}}\right.\right.}{4b^2df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$- \frac{\sqrt{-dg+ch}(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg}{4}$$

```
[Out] -1/4*(C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))-4
*b*d*f*h*(2*A*b*d*f*h-C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(b*
x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2
),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^
(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h
+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/d^2/f^2/h^3/(-a*d+b*c)^(1/2)/(d
*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(
1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b/d/f^2/h^2/(d*x+c)^(1/2)+1/2*C*(b*x+a)^(1
/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+1/4*C*(-a*f+b*e)*(a*d*f
```

```
*h+b*(c*f*h+3*d*(e*h+f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+
f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/
2))*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^
(1/2)/b^2/d/f^2/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e
*h+f*g)/(b*x+a))^(1/2)-1/4*C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*EllipticE((-
c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e
*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*
x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/b/d^2/f^2/h^2/((-
c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.56 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {1615, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\cdots\right)}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$+\frac{(be-af)\sqrt{bg-ah}(bcfh+adfh+3bd(fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\cdots\right)\right)}{4b^2df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}$$

$$+\frac{(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}C}{4bdf^2h^2\sqrt{c+dx}}$$

$$-\frac{\sqrt{ch-dg}(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg+\cdots)))}{4b^2d\cdots}$$

```
[In] Int[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] (C*(a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/(4*b*d*f^2*h^2*Sqrt[c + d*x]) + (C*Sqrt[a + b*x]*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) - (C*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*
(a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(
g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[
e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e
 - a*f)*(d*g - c*h))])/(4*b*d^2*f^2*h^2*Sqrt[((d*e - c*f)*(a + b*x))/((b*e
- a*f)*(c + d*x))]*Sqrt[g + h*x]) + (C*(b*e - a*f)*Sqrt[b*g - a*h]*(b*c*f*h
```

```
 + a*d*f*h + 3*b*d*(f*g + e*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(
Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(
b*g - a*h)))])/(4*b^2*d*f^2*h^2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e
- a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) - (Sqrt[-(d*g) + c*h]*(C*(a*d*
f*h - 3*b*(d*f*g + d*e*h + c*f*h))*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)) -
4*b*d*f*h*(2*A*b*d*f*h - C*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*h +
c*f*h))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*S
qrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g -
 c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g
) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h
))])/(4*b^2*d^2*Sqrt[b*c - a*d]*f^2*h^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1615

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f*
g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
```

```
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{4aAdfh-C(bceg+a(deg+cfg+ceh))+2(2Abdfh-C(b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+C(adfh-3b(dfg+deh+cfh))x^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{4dfh}$$

$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}} + \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{\int \frac{-C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+2bdfh(4aAdfh-C(bceg+a(deg+cfg+ceh)))-(C(adfh-3b(dfg+deh+cfh))(adfh-}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{8bd^2f^2h^2} + \frac{(C(de-cf)(dg-ch)(adfh-3b(dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8bd^2f^2h^2}$$

$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}} + \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh} + \frac{(C(be-af)(bg-ah)(bcfh+adfh+3bd(fg+eh)))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{8b^2df^2h^2} - \frac{(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg+c}{8b^2d^2f^2h^2} - \frac{\left(C(dg-ch)(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx\right.}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}}$$
$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{C\sqrt{dg-ch}\sqrt{fg-eh}(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e\cdot}}{\sqrt{fg-eh}\sqrt{c\cdot}}\right.\right.}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$- \frac{\left((C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg+c\cdots\right.}{}$$
$$+ \frac{\left(C(be-af)(bg-ah)(bcfh+adfh+3bd(fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{}{\sqrt{1+\frac{(bc-a\cdots}{de-\cdots}}}\right.}{4b^2df^2h^2(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}}$$
$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{C\sqrt{dg-ch}\sqrt{fg-eh}(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e\cdot}}{\sqrt{fg-eh}\sqrt{c\cdot}}\right.\right.}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+ \frac{C(be-af)\sqrt{bg-ah}(bcfh+adfh+3bd(fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+f\cdot}}{\sqrt{fg-eh}\sqrt{a+b\cdot}}\right.\right.}{4b^2df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$- \frac{\sqrt{-dg+ch}(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C\cdots}{}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16972 vs. 2(937) = 1874.

Time = 36.21 (sec) , antiderivative size = 16972, normalized size of antiderivative = 18.11

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. 2(854) = 1708.

Time = 5.23 (sec) , antiderivative size = 1794, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1794
default	Expression too large to display	43214

```
[In] int((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,met
hod=_RETURNVERBOSE)

[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(1/2*C/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*
e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g
*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(
A*a-1/2*C/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g/h-a/b
)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/
f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/
(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*Ellip
ticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a
/b+e/f)/(-c/d+g/h))^(1/2))+2*(A*b-1/2*C/d/f/h*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*
e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/
2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g
/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d
)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*El
lipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d
),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(C*a-1/2*C/d/f/h*(3/2
*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g/
h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*
(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(
1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-
g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/
(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c
/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a
*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/
(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(
1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**
(1/2),x)
[Out] Integral((A + C*x**2)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="giac")
[Out] integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x
)^(1/2)),x)
```

```
[Out] int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x
)^(1/2)), x)
```

3.33 $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 757

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$
$$- \frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+ \frac{(a^2Cfh+abC(fg+eh)-b^2(Ceg-2Afh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\right.}{b^2fh\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$- \frac{C\sqrt{-dg+ch}(adfh+b(dfg+deh+cfh))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\text{ar}\right.}{b^2d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -C*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x
+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b
*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d
*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/
2)/b^2/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+C*(b*x+a)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b/f/h/(d*x+c)^(1/2)+(a^2*C*f*h+a*b*C*(e*h+f*g)
-b^2*(-2*A*f*h+C*e*g))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^
(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(
(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b^2/f/h/(-a*h+b*
g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*
x+a))^(1/2)-C*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*
x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^
(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c
))^(1/2)/b/d/f/h/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2
)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1617, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\right.}{b^2fh\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$-\frac{C(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(adfh+b(cfh+deh+dfg))\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\text{ar}\right.}{b^2dfh^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$-\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}$$

$$+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

```
[In] Int[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] (C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*Sq
rt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/
((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/
(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d
*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]
*Sqrt[g + h*x]) + ((a^2*C*f*h + a*b*C*(f*g + e*h) - b^2*(C*e*g - 2*A*f*h))*
Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Ellipti
cF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])],
 -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b^2*f*h*Sqrt[b*g
 - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g
- e*h)*(a + b*x)))]) - (C*Sqrt[-(d*g) + c*h]*(a*d*f*h + b*(d*f*g + d*e*h +
c*f*h))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c
*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g)
+ c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))
])/(b^2*d*Sqrt[b*c - a*d]*f*h^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
```

```
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1617

```
Int[((A_.) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[C*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x])), x] + (Di
st[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) - C*(a*d*f*h + b*(d*f*g +
 d*e*h + c*f*h))*x, x], x], x] + Dist[C*(d*e - c*f)*((d*g - c*h)/(2*b*d*f*h
)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]
) /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}} + \frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bdfh} \\
&\quad + \frac{(C(de-cf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bdfh} \\
&= \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}} \\
&\quad + \frac{\left(a^2Cfh+abC(fg+eh)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b^2fh} \\
&\quad - \frac{(C(adfh+b(dfg+deh+cfh)))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2b^2dfh} \\
&\quad - \frac{\left(C(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}
\end{aligned}$$

$$= \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}} - \frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$- \frac{\left(C(adfh+b(dfg+deh+cfh))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int\frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad}{dg-c}}}\right.}{b^2dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$+ \frac{\left((a^2Cfh+abC(fg+eh)-b^2(Ceg-2Afh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-}}\right.}{b^2fh(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}} - \frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$+ \frac{(a^2Cfh+abC(fg+eh)-b^2(Ceg-2Afh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\right.}{b^2fh\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$- \frac{C\sqrt{-dg+ch}(adfh+b(dfg+deh+cfh))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin\right.}{b^2d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6321 vs. 2(757) = 1514.

Time = 34.84 (sec) , antiderivative size = 6321, normalized size of antiderivative = 8.35

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h
*x]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 1065, normalized size of antiderivative = 1.41

method	result	size
elliptic	Expression too large to display	1065
default	Expression too large to display	15875

```
[In] int((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*A*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))
^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*
(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x
+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+C*((x+a/b)*(x+
e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d
)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h
+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/
b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-
a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)
/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2
))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*Elliptic
Pi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f
-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x
+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*x**2+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**
(1/2),x)
```

```
[Out] Integral((A + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
 g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^
(1/2)),x)
```

```
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^
(1/2)), x)
```

3.34 $\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 867

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab^2+a^2C)\,d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2(Ab^2+a^2C)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{2(2abcC+Ab^2d-a^2Cd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b^2(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+\frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)(c}{(bc-ad)(f}\right.}{b^2\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 2*C*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+
a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h
+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)
*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/h/(-a*d+b*c)^(1/2)/(d*x+
c)^(1/2)/(f*x+e)^(1/2)+2*(A*b^2+C*a^2)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g
)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*(A*b^2+C*a^2)*(d
*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b
*x+a)^(1/2)-2*(A*b^2*d-C*a^2*d+2*C*a*b*c)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e
)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(
-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/
2)/b^2/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b
```

```
*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*(A*b^2+C*a^2)*EllipticE((-c*h+d*g)^
(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(
-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2
)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(-
a*h+b*g)/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {1619, 1616, 1612, 176, 430, 171, 551, 182, 435}

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(Ca^2+Ab^2)}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2(-Cda^2+2bcCa+Ab^2d)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b^2(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+\frac{2C\sqrt{ch-dg}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right.}{b^2\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[In] Int[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] (2*(A*b^2 + a^2*C)*d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[c + d*x]) - (2*(A*b^2 + a^2*C)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a
 + b*x]) - (2*(A*b^2 + a^2*C)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]
*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[
(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a
*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(b*(b*c - a*d)*(b*e - a*f)*(b*
g - a*h)*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x
]) - (2*(2*a*b*c*C + A*b^2*d - a^2*C*d)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e
- c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e +
 f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e
- c*f)*(b*g - a*h)))])/(b^2*(b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqr
t[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (2*C
*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-
```

```
((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(
Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*
(f*g - e*h))])/(b^2*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1619

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(
d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(d
*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(m
 + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g
*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2
 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && In
tegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{\int \frac{-a(aAdfh-aC(deg+cfg+ceh)+b(cCeg-Adfg-Adeh-Acfh))+(2a^2C(dfg+deh+cfh)+b^2(cCeg+Adfg+Adeh+Acfh)+ab(Adfh-C(deg}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$

$$= \frac{2(Ab^2+a^2C)\,d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{\int \frac{-2d(acC+Abd)f(be-af)h(bg-ah)+2Cd(bc-ad)f(be-af)h(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2bd(bc-ad)f(be-af)h(bg-ah)} + \frac{((Ab^2+a^2C)(de-cf)(dg-ch))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b(bc-ad)(be-af)(bg-ah)}$$

$$= \frac{2(Ab^2+a^2C)\,d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{C\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2} - \frac{(2abcC+Ab^2d-a^2Cd)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2(bc-ad)} - \frac{\left(2(Ab^2+a^2C)(dg-ch)\sqrt{a+bx}\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-bc+ad)x^2}{be-af}}}{\sqrt{1-\frac{(dg-ch)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{c+dx}}\right)}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$= \frac{2(Ab^2+a^2C)\,d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(Ab^2+a^2C)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)\middle|\frac{(bc-ad)(fg-e}{(be-af)(dg-c}\right.}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} + \frac{\left(2C(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g+hx}}{\sqrt{a+bx}}\right.}{b^2\sqrt{c+dx}\sqrt{e+fx}} - \frac{\left(2(2abcC+Ab^2d-a^2Cd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+}}{\sqrt{a+}}\right.}{b^2(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2(Ab^2 + a^2C)\, d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}} - \frac{2(Ab^2 + a^2C)\, \sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$$- \frac{2(Ab^2 + a^2C)\, \sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\sin^{-1}\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right.}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$- \frac{2(2abcC + Ab^2d - a^2Cd)\, \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b^2(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+ \frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-af)(d}{(bc-ad)(f}\right.}{b^2\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [A] (warning: unable to verify)

Time = 31.92 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.83

$$\int \frac{A + Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\, dx =$$

$$-\frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left(2aC(-bc+ad)h(-bg+ah)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)}{(fg-eh)(}}\right.\right.\right.}{}$$

```
[In] Integrate[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]),x]
```

```
[Out] (-2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*(2*a*C*(-(b*c) + a*d)*h*(-(b*g) + a*h)*EllipticF
[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
 + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] - A*b^2*h*(b*(d*g - c*h)
*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]]
, ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] + d*(-(b*g) +
a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x
))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) - a^2*C*h
*(b*(d*g - c*h)*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]
 + d*(-(b*g) + a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g
- e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*
h))]) + C*(b*c - a*d)*(b*g - a*h)^2*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a
*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], (
(-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/(b^2*(b*c - a*d
)*h*(f*g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)
*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. 2(794) = 1588.

Time = 7.83 (sec) , antiderivative size = 2286, normalized size of antiderivative = 2.64

method	result	size
elliptic	Expression too large to display	2286
default	Expression too large to display	33894

```
[In] int((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,met
hod=_RETURNVERBOSE)

[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*
c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)/((x
+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+
b*d*e*g*x+b*c*e*g))^(1/2)+2*(-C*a/b^2+1/b^2*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-
a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f
*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(
b*c*e*h+b*c*f*g+b*d*e*g)/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a
*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2))*(g/h-a/b)*((-g
/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e
/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+
c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF((
(-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2))+2*(C/b-1/b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(A*b^2+C*
a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g
+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/b/(a^3*d*f*h-a^2*b*
c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g
)*(A*b^2+C*a^2))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x
+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(
-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+
e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))
^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*Ellipti
cPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e
/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))-2*d*f*h*(A*b^2+C*a^2)/(a^3
*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g)*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h
+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2
)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c
^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c
/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*El
lipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/
(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/
```

```
b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)
,(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/
(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((C*x**2+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**
(1/2),x)
[Out] Integral((A + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
+ h*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
 + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^
(1/2)),x)
```

```
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^
(1/2)), x)
```

3.35 $\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 1070

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(2Ad(fg+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

$$-\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$+\frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(2Ad(fg+}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$+\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh)}{3(bc-ad)^2(be-af)^2(bg-}$$

$$-\frac{2(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh-C(d^2eg-c}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg-}}$$

```
[Out] -4/3*d*(A*b^3*(c*e*h+c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f*
h-2*C*(c*e*h+c*f*g+d*e*g))-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*(b
*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g
)^2/(d*x+c)^(1/2)-2/3*(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/
2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)+4/3*b*(A*b^3*(c*e*h+c*f*g
+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f*h-2*C*(c*e*h+c*f*g+d*e*g))
-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*
(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)^(1/2)-2/3*(3*a
*b*(A*d^2+C*c^2)*(e*h+f*g)-b^2*(2*A*d^2*e*g+A*c*d*(e*h+f*g)+c^2*(-A*f*h+3*C
*e*g))-a^2*(3*A*d^2*f*h-C*(-2*c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)))*EllipticF(
```

```
(-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*
(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*
x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(3/2)/(-e*h+f*
g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)+4/3*(
A*b^3*(c*e*h+c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f*h-2*C*(c
*e*h+c*f*g+d*e*g))-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*EllipticE(
(-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(
-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(
b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/(-a*d+b*c)^2/(-
a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+
g)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.32 (sec) , antiderivative size = 1070, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1619, 1613, 1616, 12, 176, 430, 182, 435}

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$+\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-}{3(bc-ad)^2(be-af)^2(bg}$$

$$-\frac{2(-((3Ad^2fh-C(-2fhc^2-dfgc-dehc+d^2eg))a^2)+3b(Cc^2+Ad^2)(fg+eh)a-b^2((3Ceg-Afh)c}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{}}$$

$$+\frac{4b(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))c}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$-\frac{4d(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

```
[In] Int[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
,x]
```

```
[Out] (-4*d*(A*b^3*(d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*
b*(3*A*d*f*h - 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2*(2*A*d*(f*g + e*h) - c*
(3*C*e*g - 2*A*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c -
a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[c + d*x]) - (2*(A*b^2 + a^2*C)*Sqrt
[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(b*e - a*f)*(b*g - a*
h)*(a + b*x)^(3/2)) + (4*b*(A*b^3*(d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g +
d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2*(2
*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^2*Sqrt[a + b*x]) + (4
```

```
*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(A*b^3*(d*e*g + c*f*g + c*e*h) + a^3*C*(d*
f*g + d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g + c*f*g + c*e*h)) - a*
b^2*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e
 - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c
*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*
h))/((b*e - a*f)*(d*g - c*h))])/(3*(b*c - a*d)^2*(b*e - a*f)^2*(b*g - a*h)^
2*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2
*(3*a*b*(c^2*C + A*d^2)*(f*g + e*h) - b^2*(2*A*d^2*e*g + A*c*d*(f*g + e*h)
+ c^2*(3*C*e*g - A*f*h)) - a^2*(3*A*d^2*f*h - C*(d^2*e*g - c*d*f*g - c*d*e*
h - 2*c^2*f*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt
[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/
(3*(b*c - a*d)^2*(b*e - a*f)*(b*g - a*h)^(3/2)*Sqrt[f*g - e*h]*Sqrt[c + d*x
]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 176</u>

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

<u>Rule 182</u>

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

<u>Rule 430</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1619

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(
d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(d
*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(m
 + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g
*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2
 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && In
tegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{\int \frac{-2Ab^2(deg+cfg+ceh)-3ab(cCeg-Adfg-Adeh-Acfh)-a^2(3Adfh-C(deg+cfg+ceh))+(2a^2C(dfg+deh+cfh)+b^2(3cCeg-Adfg-Adeh-}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$

$$= -\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+}} + \frac{\int b(bceg-a(deg+cfg+ceh))(2a^2C(dfg+deh+cfh)+b^2(3cCeg-Adfg-Adeh-Acfh)+3ab(Adfh-C(deg+cfg+ceh)))+a(adfh-b(d}{}$$

$$= -\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+}} + \frac{\int -\frac{2bdf(be-af)h(bg-ah)(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh-C(d^2eg-cdfg-cde}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6bd(bc-ad)^2f(be-af)^2h(bg-ah)^2} - \frac{(2(de-cf)(dg-ch)(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+}{3(bc-ad)^2(be-af)^2(b}$$

$$= -\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+}} - \frac{(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh-C}{3(bc-ad)^2(be-af)(bg-ah)} + \frac{\left(4(dg-ch)(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ce\right.}{3(bc-ad)^2(be-af)}$$

$$= -\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a}} + \frac{4\sqrt{dg-ch}\sqrt{fg-eh}(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg-}{3(bc-ad)^2(be} - \frac{\left(2(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh\right.}{3(bc-ad)^2(be-af)(bg}$$

$$= -\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c}} - \frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} + \frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a}} + \frac{4\sqrt{dg-ch}\sqrt{fg-eh}(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg-}{3(bc-ad)^2(be} - \frac{2(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh-}{3(bc-ad)^2(be-af)(bg-ah)^{3/}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11363 vs. 2(1070) = 2140.

Time = 40.54 (sec) , antiderivative size = 11363, normalized size of antiderivative = 10.62

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. 2(998) = 1996.

Time = 10.35 (sec) , antiderivative size = 3342, normalized size of antiderivative = 3.12

method	result	size
elliptic	Expression too large to display	3342
default	Expression too large to display	106972

```
[In] int((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2/3/b^2/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*
g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)*(b*d*f*h*x^4
+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*
d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x
+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+4/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^
2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h
-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(
3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*b^3*c*e*h
+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-2*C*a^2*b*c*e*
h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/((x+a/b)*(b*d*f*h*x^3+b*
c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(
1/2)+2*(C/b^2-1/3/b^2*(3*A*a*b^2*d*f*h-A*b^3*c*f*h-A*b^3*d*e*h-A*b^3*d*f*g+
3*C*a^3*d*f*h-C*a^2*b*c*f*h-C*a^2*b*d*e*h-C*a^2*b*d*f*g)/(a^3*d*f*h-a^2*b*c
*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)
+2/3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e
*g)*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*b^3*
c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-2*C*a^2*b
*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/(a^3*d*f*h-a^2*b*c*
f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^
2-2/3*(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*(3*A*a^2*b*d*f*h-2*A*a
*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*
e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C
*a^2*b*d*e*g+3*C*a*b^2*c*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+
a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/
b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(-2/3*(
a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2
*d*e*h-2*A*a*b^2*d*f*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^
3*d*e*h+C*a^3*d*f*g-2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b
^2*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*
```

```
c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2-2/3*(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f
*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-
b^3*c*e*g)^2*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f
*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-
2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g))*(g/h-a/b)
*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f
)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(
-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*
EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b
)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-
g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/
f)/(-c/d+g/h))^(1/2)))-4/3*b*d*f*h*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b
^2*d*e*h-2*A*a*b^2*d*f*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*
a^3*d*e*h+C*a^3*d*f*g-2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a
*b^2*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^
2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+
a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d
-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/
b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1
/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e
/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h
+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a
/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c
/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F]

$$\int \frac{A + Cx^2}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cx^2 + A}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d)*
f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*e +
 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c + a
^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^3 + (
(3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a^2*b*c
 + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*g)*x),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**
(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
 + g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
 + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^
(1/2)),x)
```

```
[Out] int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^
(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```